

# SPECTRUM OF MULTI-BUNCH POSITION MODEL AND PARAMETER ACQUISITION ALGORITHM\*

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## Abstract

Based on the spectrum of turn-by-turn model for the storage ring, spectrum of multi-bunch position model was derived through some assumptions. Spectrum of excited electron beam position was analyzed in Shanghai Synchrotron Radiation Facility(SSRF) and Genetic Algorithm was used to obtain the model parameters when fitting multi-curve data. Results show that, after 100 times iteration, all the correlation of fitted data and original data can be up to 95%, and the model can accurate estimate a bimodal split of the spectrum curve.

## INTRODUCTION

The emergence of the third generation of the light source, marks the synchrotron radiation light is developing to the direction of high-energy and high luminance. Stable high flow intensity and bunch instability is the focus of concern in the the beam diagnostic study. A spectrum curve of bunch position with bimodal split was found in SSRF when the storage ring ran at multi-bunch mode. Establishing an appropriate bunch position spectrum model is necessary for further study of bunch instability and other parameters of the ring accelerator storage ring.

Model parameters acquisition method is the first problem to be solved after modeling. The genetic algorithm(GA) is an optimization method put forward by American professor HOLLAND in 1975, which simulates biological evolution in the natural selection mechanism according to the theory of biological evolution and genetic variation principle. Compared with other optimization methods, GA has the advantage of global parallel search, not easy to fall into local optimum, and has no special requirements to the optimize function itself, neither continuous nor differentiable [1]. Therefore, the genetic algorithm is especially suitable to solve the optimal solution of complex nonlinear and multidimensional space.

## MULTI-BUNCH SPECTRUM MODEL

### Spectrum Function of Single Bunch Mode

To accurately determine tune value of the accelerator, a suitable spectrum model of bunch position has been established for storage ring running at single bunch mode in the previous. And the relationship between tune value and beam current was found.

Literature [2] shows that the spectrum function of the

bunch position in single bunch mode can be expressed as:

$$f(x) = \begin{cases} \frac{a-d}{\sqrt{1+b^2\left(\frac{2c-x}{c}-\frac{c}{2c-x}\right)^2}} + d, (x < c) \\ \frac{a-d}{\sqrt{1+b^2\left(\frac{x-c}{c}-\frac{c}{x}\right)^2}} + d, (x \geq c) \end{cases} \quad (1)$$

Wherein: a is the curve peak; b represents the curve width, the characterization of the quality factor Q of the vacuum chamber; c is the frequency value at the peak of the curve in the observed range; d is the offset for noise.

When the spectral curve within the observation range is expanded to the entire band, the formula (1) should be translational as a whole, i.e., replace x with x-f. So the spectrum function is:

$$f(x) = \begin{cases} \frac{a-d}{\sqrt{1+b^2\left(\frac{2c-x+f}{c}-\frac{c}{2c-x+f}\right)^2}} + d, (x < c+f) \\ \frac{a-d}{\sqrt{1+b^2\left(\frac{x-f}{c}-\frac{c}{x-f}\right)^2}} + d, (x \geq c+f) \end{cases} \quad (2)$$

### Spectrum Function Modeling of Multi-bunch Mode

Considering parameters b, c, f will be related to bunch charge, for simplicity, suppose b, c, f and bunch charge are a linear relationship.

In the injection process of electron storage ring, there are only two kinds of charge amount bunches at the same time from a macro point of view. And assume that each bunch wakefield effects the entire storage ring. Thus in the operating mode of the multi-beam group, the spectrum function of the bunch position can be regarded as the combined effect caused by two interacting single bunch.

Assume there are two kind of bunches with charge quantity q1, q2 number n1, n2 in the storage ring, and q1 > q2.

Suppose two single-bunch interaction does not affect parameters f, c, and only affects the curve width b and peak a.

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For the curve width:

The front has the assumption that b and single bunch charge are a linear relationship.

$$\text{Introduce charge quantity: } dq = \frac{q1 * q2}{q1 + q2}$$

To calculate b of q1, using charge quantity

$$q1' = q1 - dq * \frac{n2}{n1 + n2}$$

To calculate b of q2, using charge quantity

$$q2' = q2 + dq * \frac{n1}{n1 + n2}$$

Curve peak is affected as follow:

$$\frac{\alpha(q2)}{\alpha(q1)} = \text{rate}^3$$

Wherein rate is a linear function of n1 and n2. While n1 + n2 is a constant when storage ring working, rate is actually a linear function of n1.

Taking all these factors, the spectrum function model in multi-bunch mode can be expressed as:

$$f(x) = f_{q1}(x) + f_{q2}(x) \quad (3)$$

$$f_{qi}(x) = \begin{cases} \frac{ai}{\sqrt{1 + \beta(qi)^2 \left( \frac{2d(qi) - x + f(qi)}{d(qi)} - \frac{d(qi)}{2d(qi) - x + f(qi)} \right)^2}}, & (x < d(qi) + f(qi)) \\ \frac{ai}{\sqrt{1 + \beta(qi)^2 \left( \frac{x - f(qi)}{d(qi)} - \frac{d(qi)}{x - f(qi)} \right)^2}}, & (x \geq d(qi) + f(qi)) \end{cases} \quad (4)$$

$i=1,2$

$$b(q) = b0 + b1 * q \quad (5)$$

$$c(q) = c0 + c1 * q \quad (6)$$

$$f(q) = f0 + f1 * q \quad (7)$$

$$a2 = (r0 + r1 * n1)^3 * a1 \quad (8)$$

And need to obtain the parameters b0, b1, c0, c1, f0, f1, r0, r1, a1.

In formula (4), noise offset has been ignored.

## PARAMETER ACQUISITION ALGORITHM

The above model has a total of nine parameters. Since a1 is related to the intensity of the excitation to the beam, normalization processing can be carried out. So in fact, we need to obtain the eight parameters. But formula (2) and formula (4) are nonlinear functions, when curve fitting, it's very easy to fall into local optimum that we can not get the correct parameters. Further, after getting the value of each parameter, the model should accord with all the conditions of q1, q2, N1, N2 during the injection process of electron storage ring, namely multiple curve fitting should be applied in the acquisition of eight parameters.

For these reasons, we use genetic algorithm to obtain the parameters of the model. Program flow chart is shown in Figure 1.

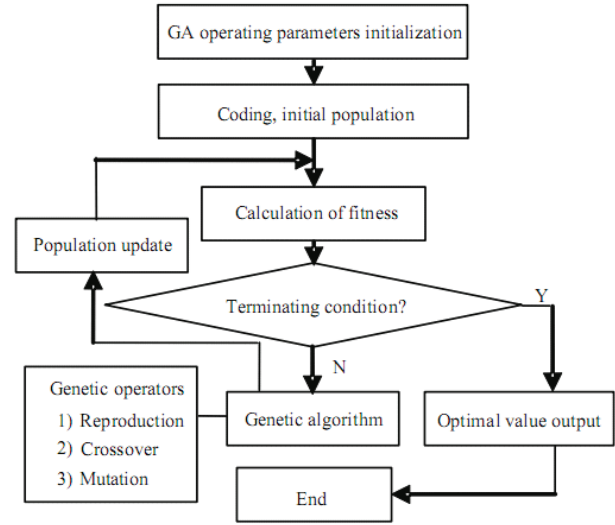


Figure 1: GA program flow chart.

## BEAM EXPERIMENT

To verify the effectiveness of the model and parameters acquisition algorithm, we analyse all the spectrum data of beam position during the injection process of SSRF.

### Data Fitting Results

Figure 2 is comparison curve given by the measured spectrum and model at the beginning of injection.

Figure 3, Figure 4 are comparison curve given by the measured spectrum and model when there are two kinds of bunch in the storage ring.

Figure 5 is comparison curve given by the measured spectrum and model at the end of injection.

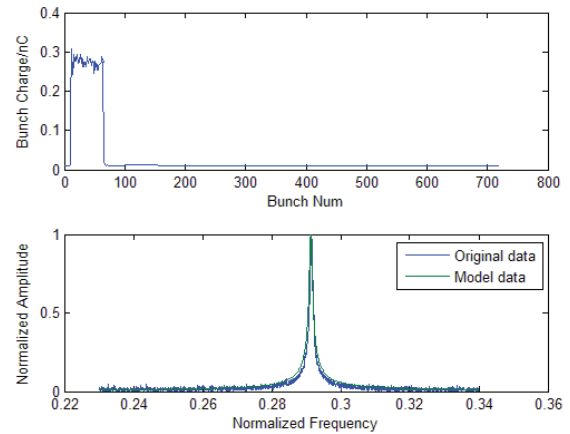


Figure 2: Distribution of bunch in the storage ring(up) and spectrum curve of multi-bunch position(down) at the beginning of injection.

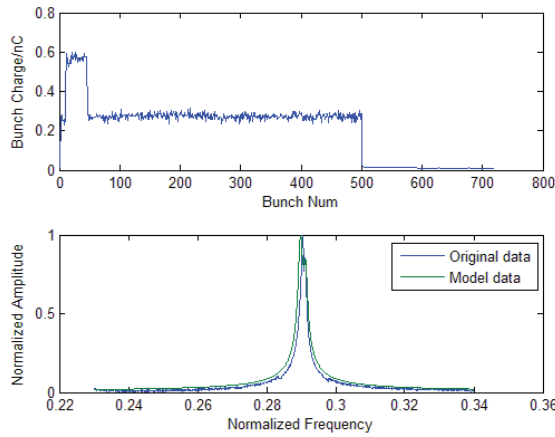


Figure 3: Distribution of bunch in the storage ring(up) and spectrum curve of multi-bunch position(down) when there are two kinds of bunch in the storage ring.

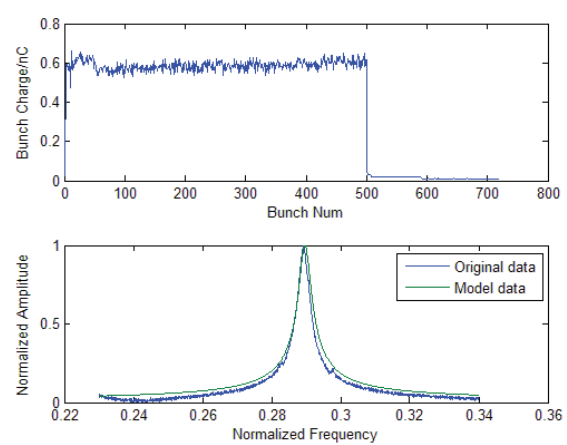


Figure 5: Distribution of bunch in the storage ring(up) and spectrum curve of multi-bunch position(down) at the end of injection.

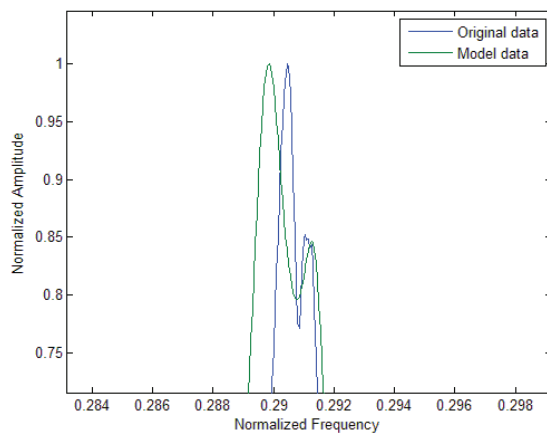


Figure 4: Enlargement of spectrum curve of multi-bunch position on the peak in Figure 3.

### Evaluate Fitting Results

From the curve comparison result, the curve given by The model and the actual are agreement. And fine structure of bimodal split can be seen when there are two kinds of bunch. To further assess the effect of data fitting, Figure 6 shows correlation evaluation between the spectral curves given by model and all valid measurement spectrum data of bunches position. All the correlation of fitted data and original data are up to 95%.

### CONCLUSION

Under the assumptions of two clustering bunches and the bunch wakefield role area for the entire storage ring, based on the spectrum of turn-by-turn model, spectrum of multi-bunch position model was derived. Using genetic algorithm to obtain the model parameter values has the

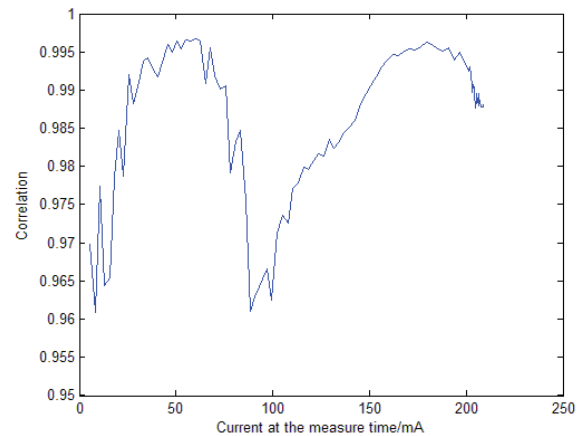


Figure 6: Evaluation by correlation.

advantage of initial value less demanding, not easy to fall into local optima. Beam experiment shows the model accords with the actual and the genetic algorithm can accurately obtain the model parameters.

### REFERENCES

- [1] Lance D. Chambers , *The Practical Handbook of Genetic Algorithms: Applications, Second Edition*, (New York: Chapman and Hall, 2000), 80.
- [2] Yang Guisen, Leng Yongbin, Yuan Renxian, Yan Yingbing, Beam instabilities based on spectrum of turn-by-turn position. *High Power Laser and Particle Beams*, 08(2011)23.