

Femtosecond Resolution Bunch Profile Diagnostics

Bernhard Schmidt
IBIC12, Tsukuba, Japan

driving forces

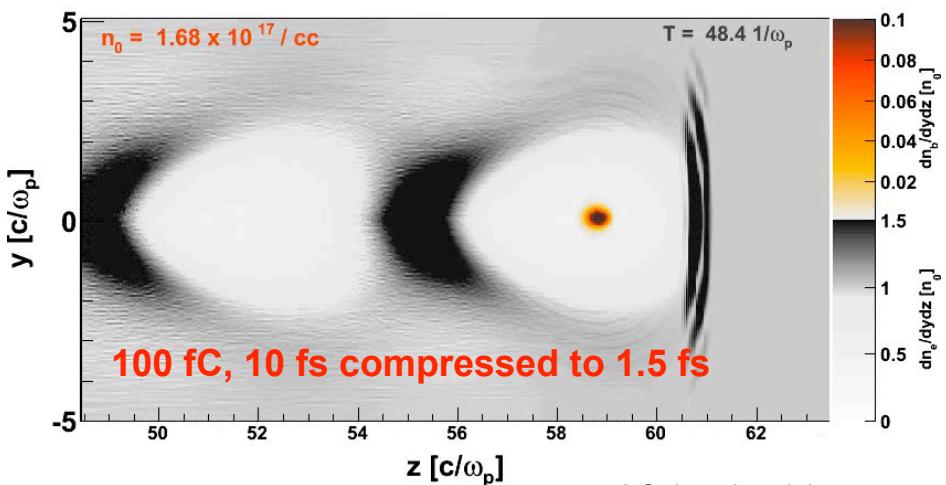
Intro

fs bunches driving fs x-ray pulses :

below ps & nC
↓
few fs & few pC

plasma wakefield experiments :

the fs & fC challenge



J. Grebenyuk *et al.*, in preparation

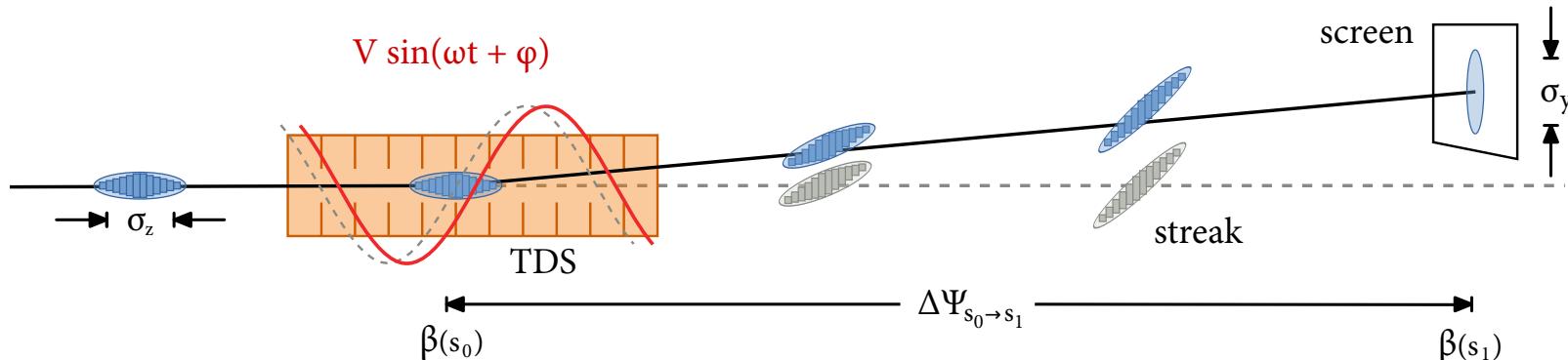
- status of established technologies
- progress towards online monitoring
- exciting experiments
- new and demanding ideas

Transverse Deflecting Structures (TDS)

TDS

well established - versatile - high resolution - bulky - expensive

RF - transverse deflector at zero crossing



imprint $y'(s)$ → $R_{y'}$ → measure $y(s)$

$$R_t = \sqrt{\frac{\epsilon}{\beta_0}} \frac{E_b}{eV\omega_{RF} \sin(\Delta\Psi)}$$

time resolution

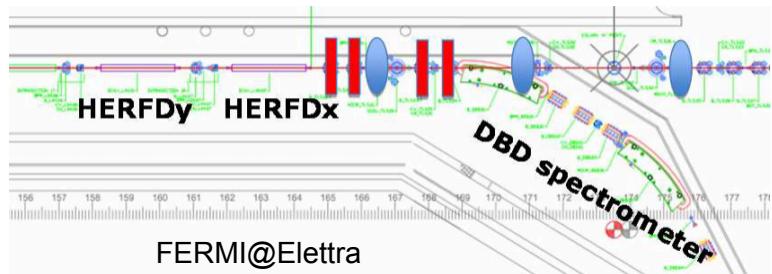
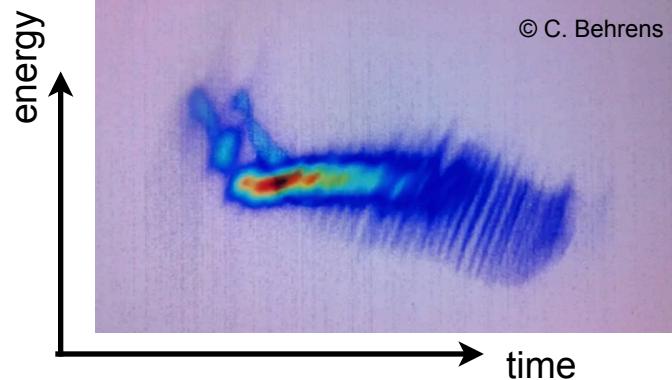
limited by beam emittance
streak power and RF-frequency
beam optics

longitudinal phase space

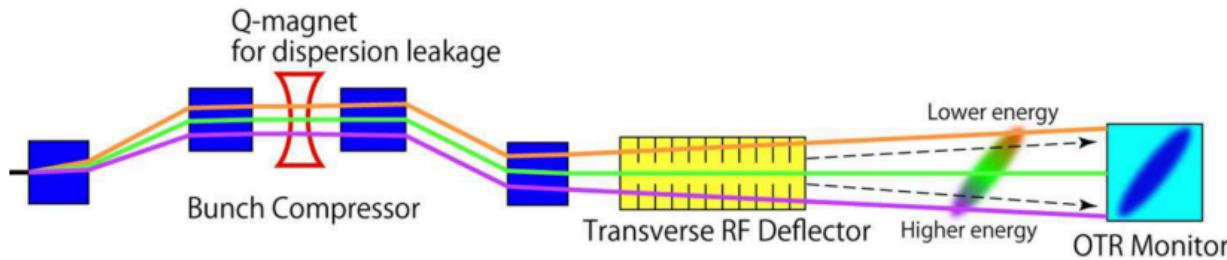
TDS

combine TDS with orthogonal dispersion

FLASH, LCLS, FERMI: TDS + Dipol spectrometer



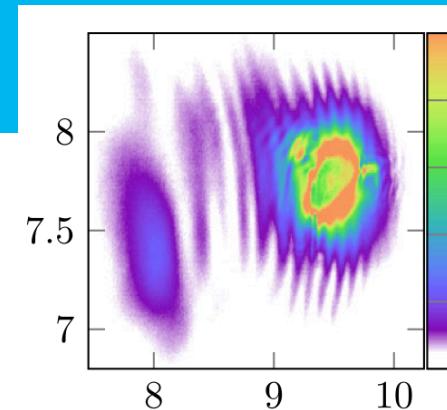
SACLA: TDS + „dispersion-leakage“
from compressor



see H. Maesaka: MOIC02

imaging problems

TDS imaging suffers from COTR due to micro-bunching



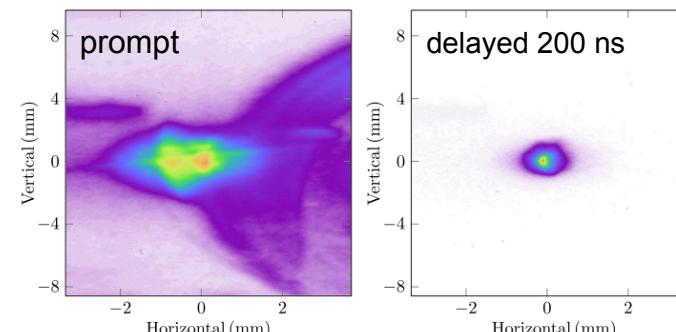
TDS

replace metallic screen by scintillator
→ still COTR from surface

mask central OTR cone

S. Matsubara
MOCC04

delayed image using gated camera



C. Behrens et al. , PRSTAB 15, 062801 (2012)

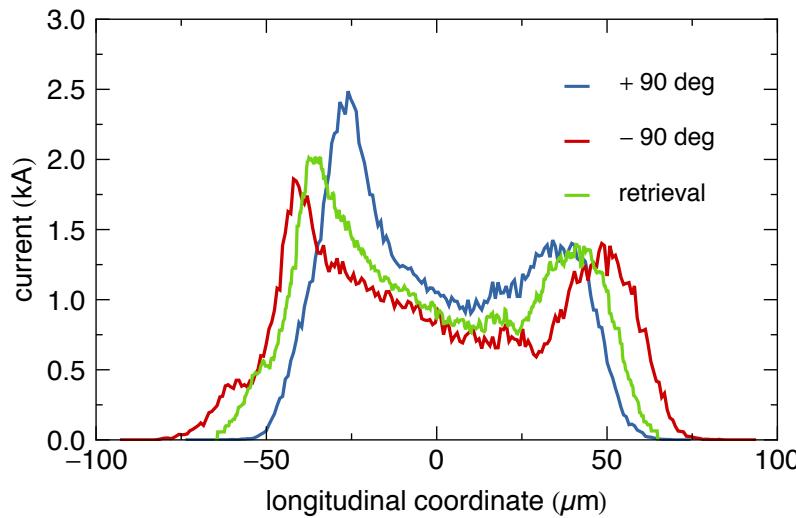
use dispersive arm (FLASH, LCLS..)
→ but no operation parallel to FEL

- disregard imaging, use wire scanners
(LCLS)

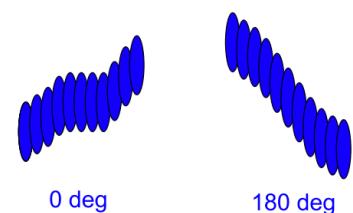
- induces energy spread
- wakefields → minor effects for high energies

non-vanishing problem : intrinsic (y' -z) correlations

retrieval from 2 streak polarities (ref. H. Loos)
(substantially different bunch profiles)



data from FLASH © S. Wesch



*bias free measurements
need at least two
opposite streaks*

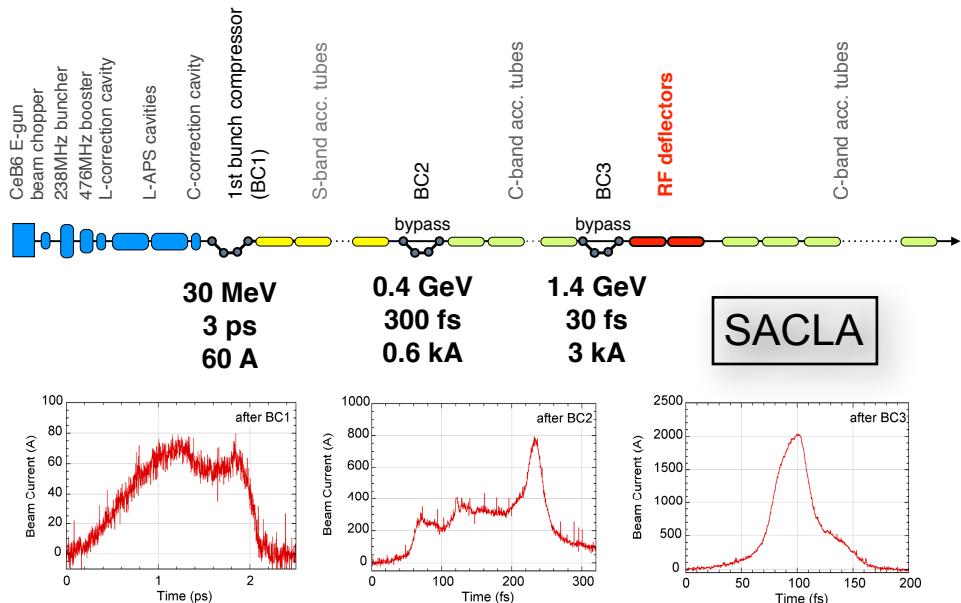
att.: TDS is not longer single shot

way to the future

TDS

machine	f-RF	V_{\max}	E_{beam}	resolution
FLASH	2.8 GHz (S)	25 MV	1.2 GeV	~10 fs
FERMI	3.0 GHz (S)	20 MV	1.2 GeV	< 20 fs
SACLA	5.7 GHz (C)	60 MV	1.4 GeV	~ 10 fs
XTCAV@LCLS	11.4 GHz (X)	46 MV	< 13.6 GeV	~1 fs exp.

Bunch length measurements with RF deflector



K. Togawa et al., Proceedings FEL2011, p.259

H. Maesaka: MOIC02

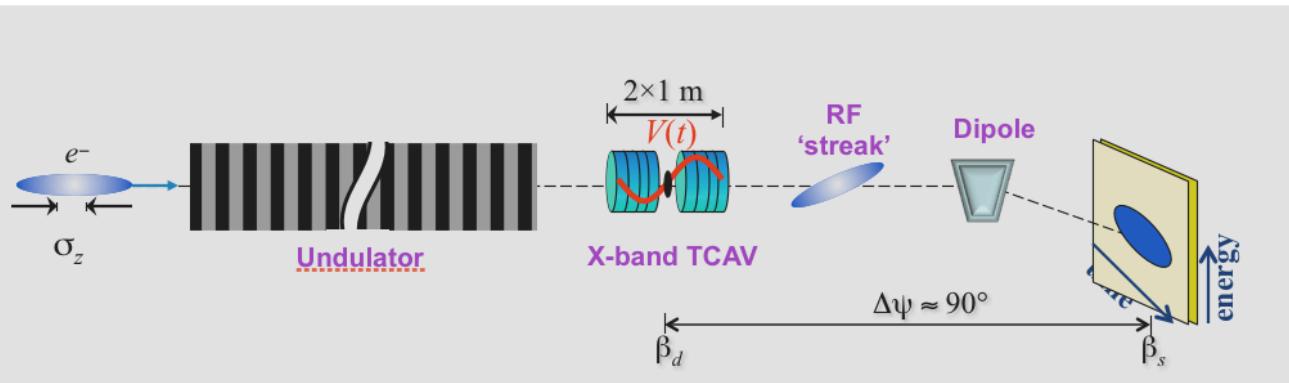
time resolution ~10 fs limited by beam size at screen

the XTCAV project at LCLS

TDS

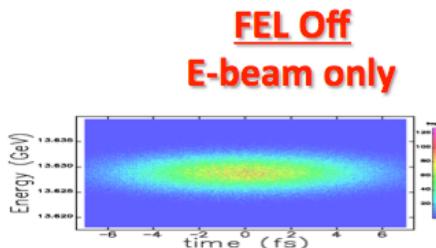
XTCAV + spectrometer behind undulator

P. Krejcik, TUPA41

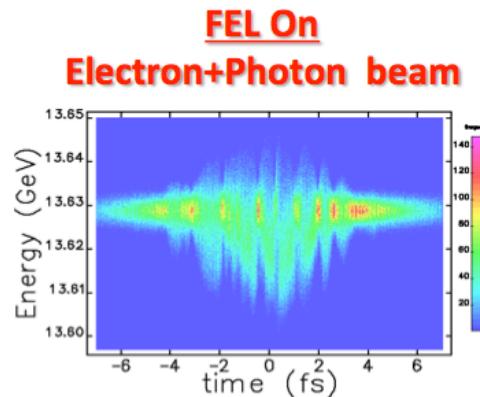


derive photon pulse profile
from long. phase space measurement

expected performance



© P. Krejcik et al.



status : structures installed
RF-power next spring

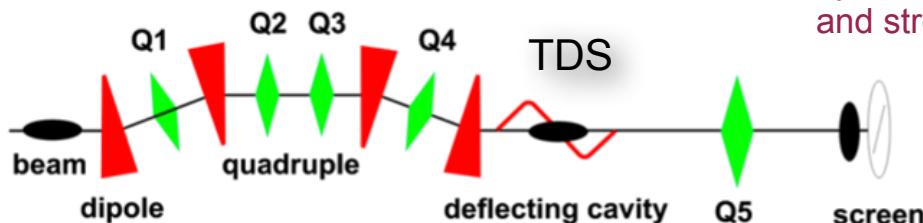
beating the limits : new ideas

TDS

„the ideal z-mapper“

D. Xiang, Y. Ding

Phys. Rev. ST Accel. Beams 13, 094001 (2010)



by careful adjusting the chicane parameters
and streaking strength of TDS

$$y(s) = f k z$$
$$R = R_b R_s = \begin{bmatrix} 0 & 0 & fk & 0 \\ 1/f & -R_{12}/f & 0 & 1/fk \\ 0 & 1/k & 1 & 0 \\ -k & kR_{12} & 0 & 0 \end{bmatrix}.$$

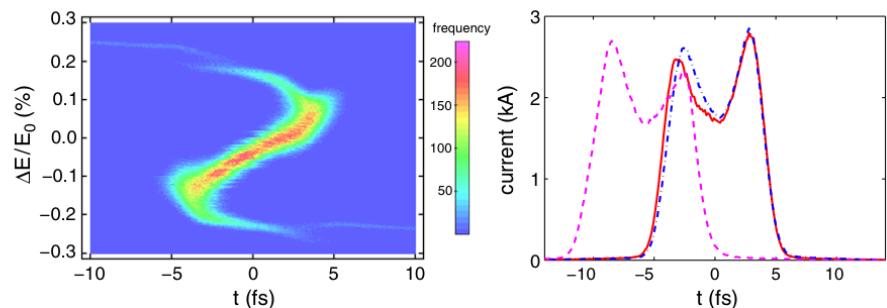
FIG. 1. Scheme for longitudinal-to-transverse mapping.

- + free from correlation problems
- + resolution limited by second order terms

- resolution scales with δE^2
- bulky, complex, needs tight balancing

ex: 0.2 fs for $\delta E=0.1\%$

simulation result



optical - streaking

Longitudinal profile diagnostic scheme with subfemtosecond resolution for high-brightness electron beams

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¹Department of Physics and Astronomy, UCLA, Los Angeles, California 90095, USA

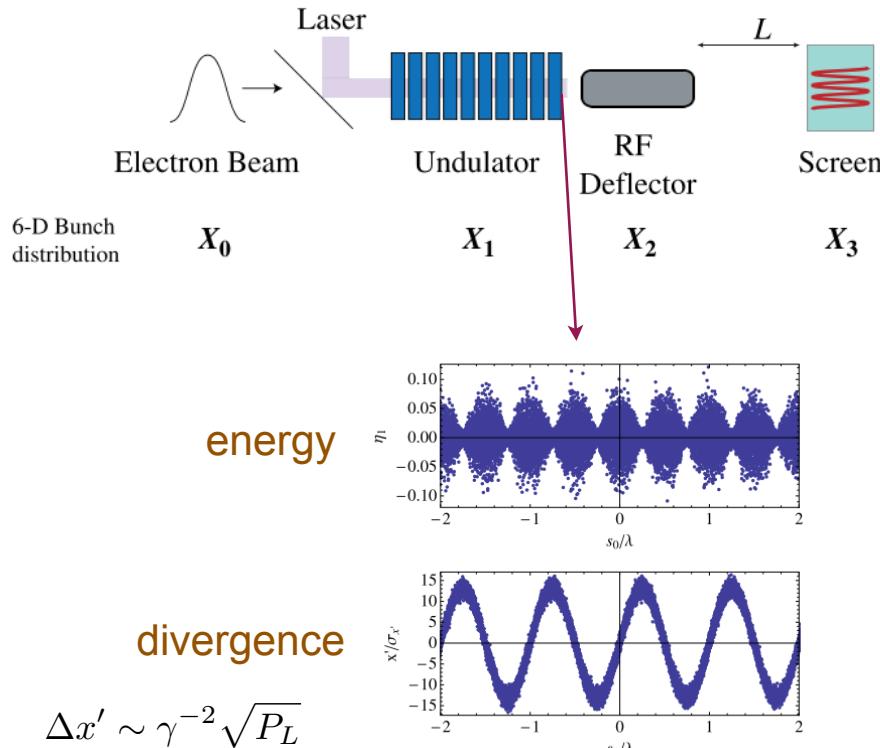
²RadiaBeam Technologies, Santa Monica, California 90404, USA

³SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA

⁴Department of Electrical Engineering, UCLA, Los Angeles, California 90095, USA

(Received 10 March 2011; published 13 July 2011)

laser + TDS : high resolution + large dynamic range



divergence

$$\Delta x' \sim \gamma^{-2} \sqrt{P_L}$$

high beam energy \rightarrow GW (TW) laser power

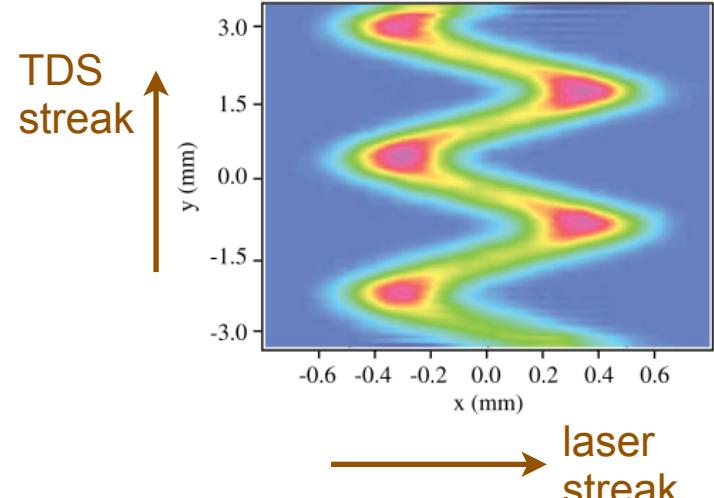


TABLE I. Simulation parameters for the UCLA Neptune case.

Beam energy	E	12.9 MeV
Normalized emittance	ϵ_n	1.0 mm mrad
Energy spread	σ_γ	5×10^{-3}
Undulator peak field	B_0	1.02 T
Undulator period	λ_u	9 mm
Undulator length	L_u	9 cm
Undulator parameter	K	0.855
Laser wavelength	λ	10.6 μ m
Laser waist	w_R	250 μ m
Laser power	P_L	300 MW

CO₂ laser

proposed proof of concept
experiment at UCLA

Coherent Radiation

CR

Spectral distribution of „bunch radiation“ with $N_e \gg 1$

For all **direct** radiation processes : incoherent and **coherent** contribution

Synchrotron -

Edge -

Transition - Radiation

Diffraction -

..

$$\frac{dU}{d\lambda d\Omega} = \left(\frac{dU}{d\lambda d\Omega} \right)_1 (N + N^2 |F_{3d}(\lambda, \Omega)|^2)$$

single electron
spectrum

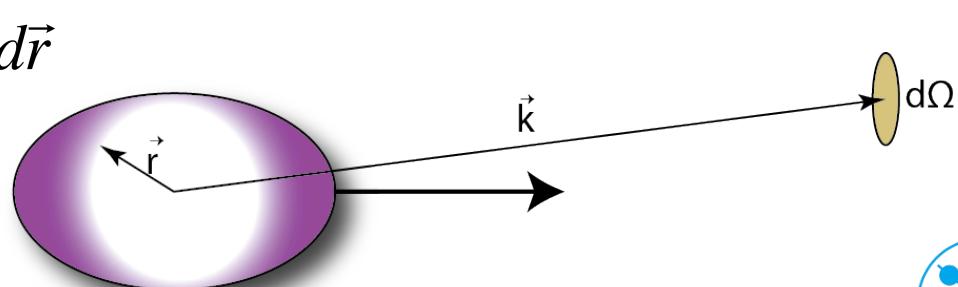
Form factor
reveals charge distribution

wavelength space

configuration space

$$F_{3D}(\vec{k}) = \int S_{3D}(\vec{r}) e^{-i\vec{k}\vec{r}} d\vec{r}$$

$$|\vec{k}| = \frac{2\pi}{\lambda}$$



$$F_{3D}(\vec{k}) = F(\lambda) F_T(\lambda, \theta)$$

*IF no long. - trans. correlation : factorize
 F_T for fixed θ , only weak λ dependence*

complex function

attn.: fails for $\sigma_r \geq \gamma\lambda/(2\pi)$



$$F(\lambda) = \int_{-\infty}^{\infty} S(z) e^{-2\pi iz/\lambda} dz \quad \textbf{longitudinal form factor}$$

coherent radiation intensity

$$\frac{dU}{d\lambda d\Omega} \approx \left(\frac{dU}{d\lambda d\Omega} \right)_1 N^2 |F(\lambda)|^2$$

$\left(\frac{dU}{d\lambda d\Omega} \right)_1$ single electron contribution

$$F(\lambda) = |F(\lambda)| e^{-\phi(\lambda)}$$

notice : the **phase** is missing !

measuring techniques

CR

Lai, Happek, Sievers (1994)

interferometer : scanning device, simple (one channel readout)..

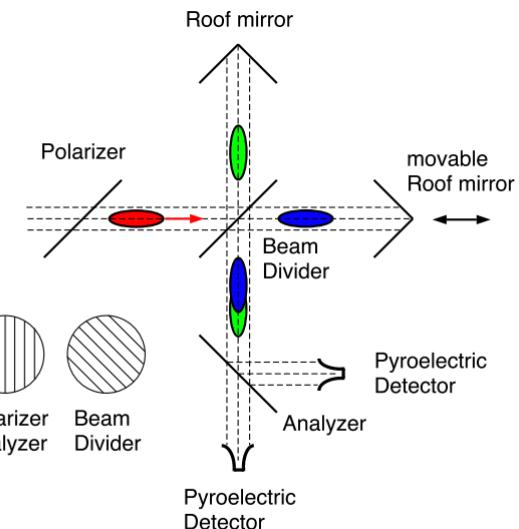
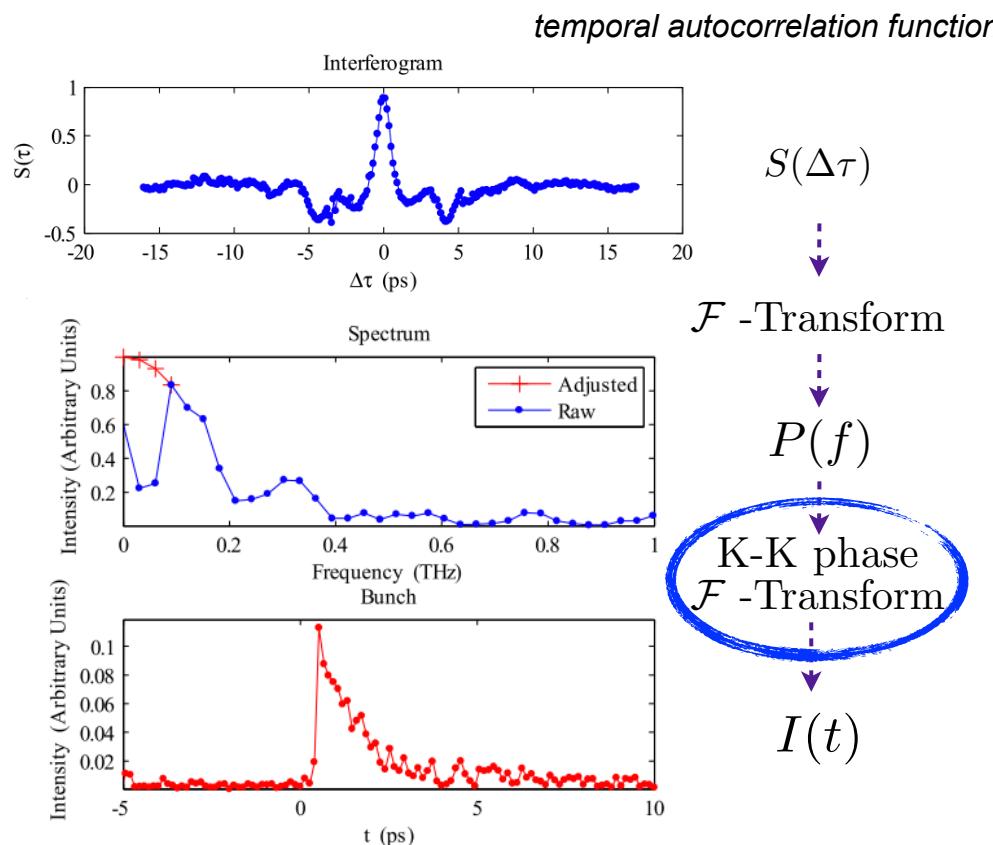


Figure 1: The Martin Puplett interferometer.

© B. Leißner et al., PAC1999

- not single shot
- diffraction limits resolution

single shot spatial auto correlator

CR

REVIEW OF SCIENTIFIC INSTRUMENTS **83**, 043302 (2012)

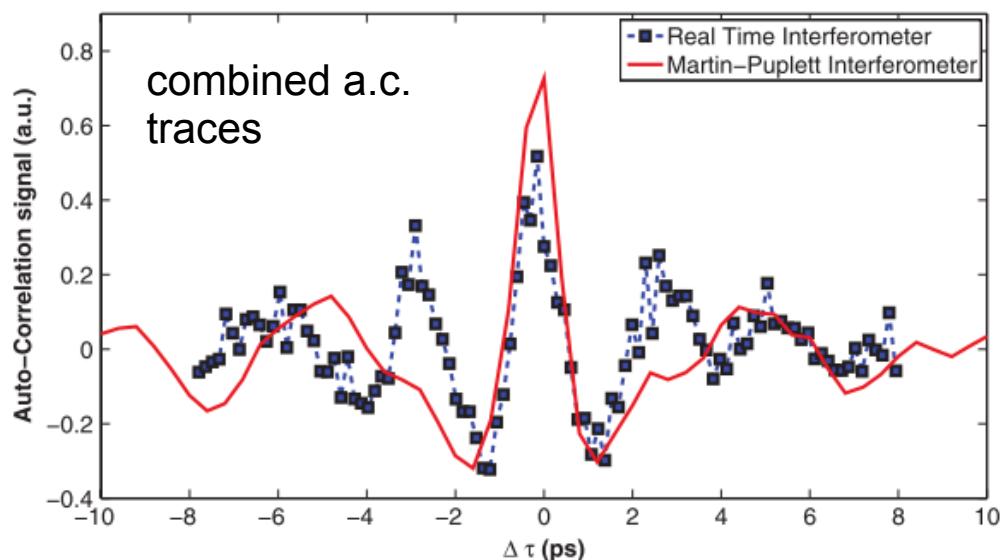
Demonstration of a real-time interferometer as a bunch-length monitor in a high-current electron beam accelerator

J. Thangaraj,^{1,a)} G. Andonian,^{2,3} R. Thurman-Keup,¹ J. Ruan,¹ A. S. Johnson,¹ A. Lumpkin,¹ J. Santucci,¹ T. Maxwell,¹ A. Murokh,³ M. Ruelas,³ and A. Ovodenko³

¹Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

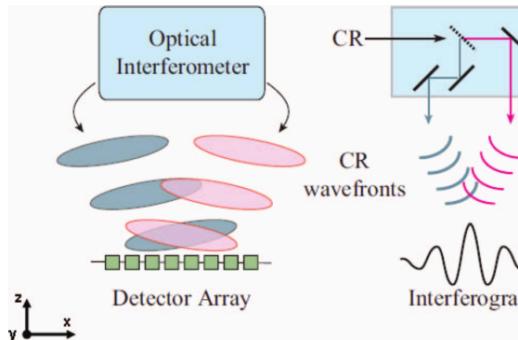
²Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA

³RadiaBeam Technologies, Santa Monica, California 90404, USA

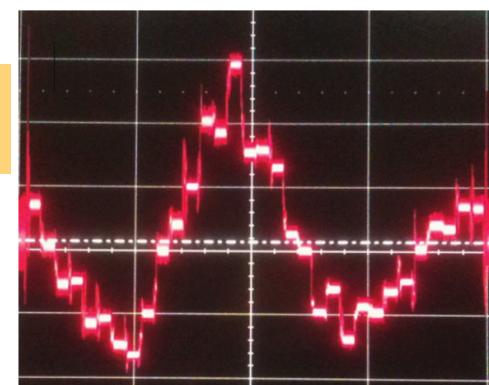


data taken at A0 photoinjector (Fermilab)

resolution determined by transverse beam size
and mixing angle



G. Andonian
TUPA39



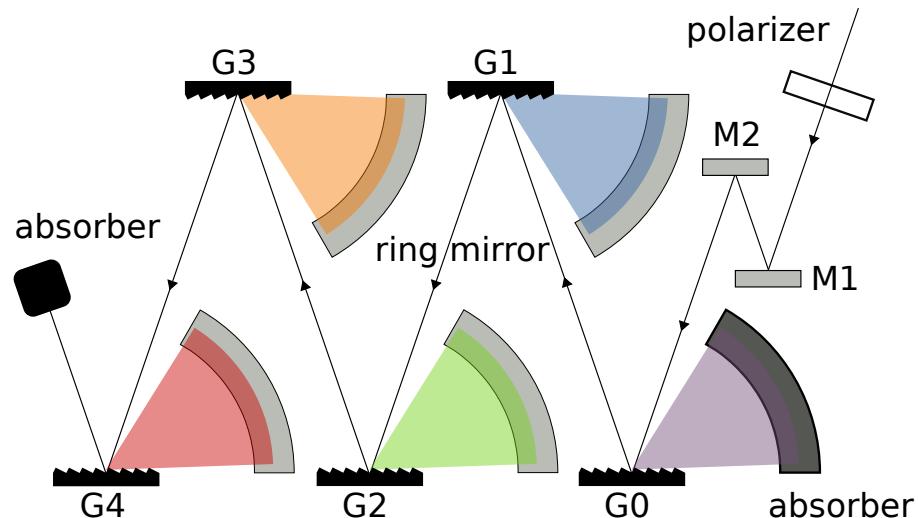
CRISP4 : four stage grating spectrometer

CRD

diffractive grating + parallel read out : single shot spectrometer

but : higher order problem ! $\lambda/2$ in 2. order and λ in 1. order have same diffraction angle

- pre-filter short wavelengths
- only factor 2 in λ with one grating possible , needs cascaded grating stages



4 stages : 1 decade in λ (parallel)

2 x 4 stages : 2 decades in λ (sequentially)

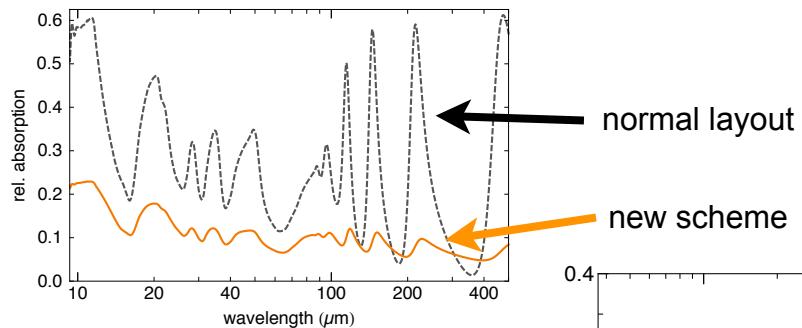
monitoring : relative intensities ok

form factor :
absolute intensity as function of λ required ! Needs detailed understanding of all components

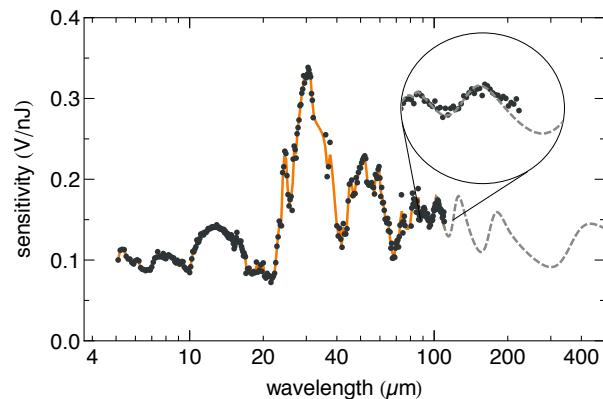
installation & calibration

CRD

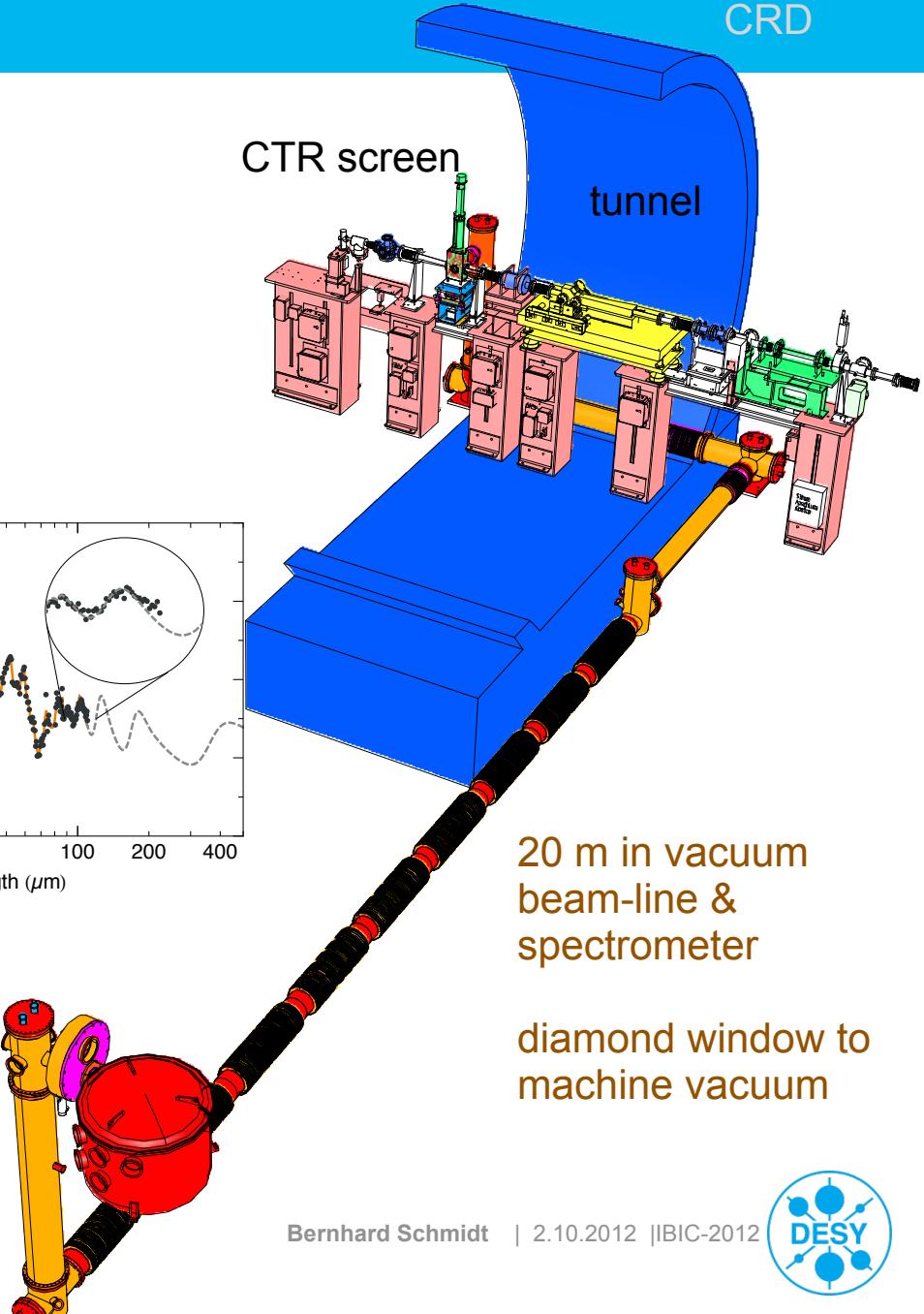
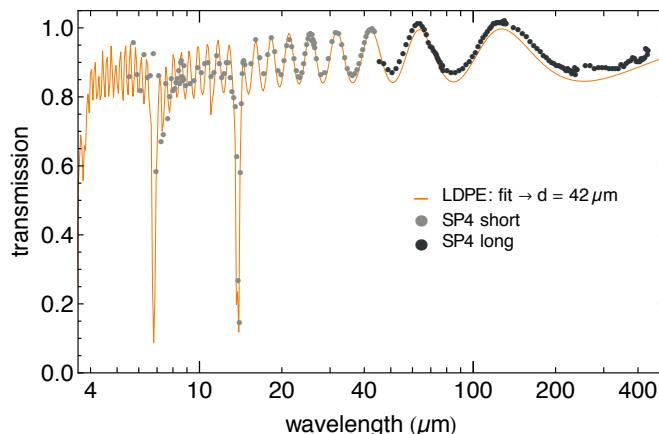
absolute $|F|$ measurement needs
full understanding of all details



120 channels pyroelectric
detectors



wavelength calibrated..



Nuclear Instruments and Methods in Physics Research A 665 (2011) 40–47

A multi-channel THz and infrared spectrometer for femtosecond electron bunch diagnostics by single-shot spectroscopy of coherent radiation

Stephan Wesch*, Bernhard Schmidt, Christopher Behrens, Hossein Delsim-Hashemi, Peter Schmüser

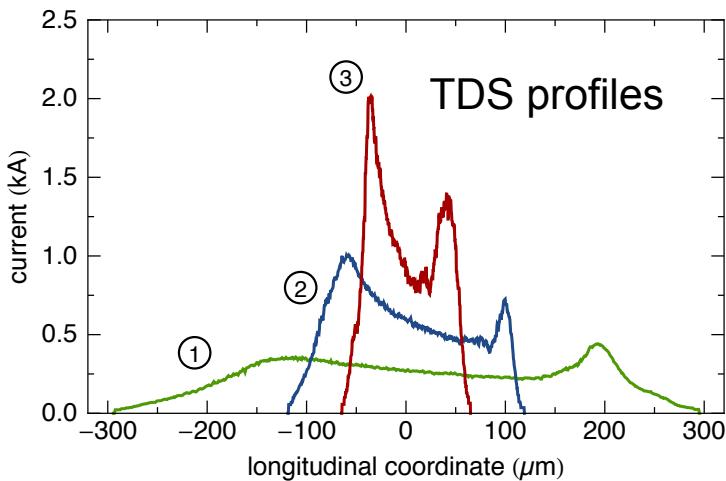
Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, 22607 Hamburg, Germany



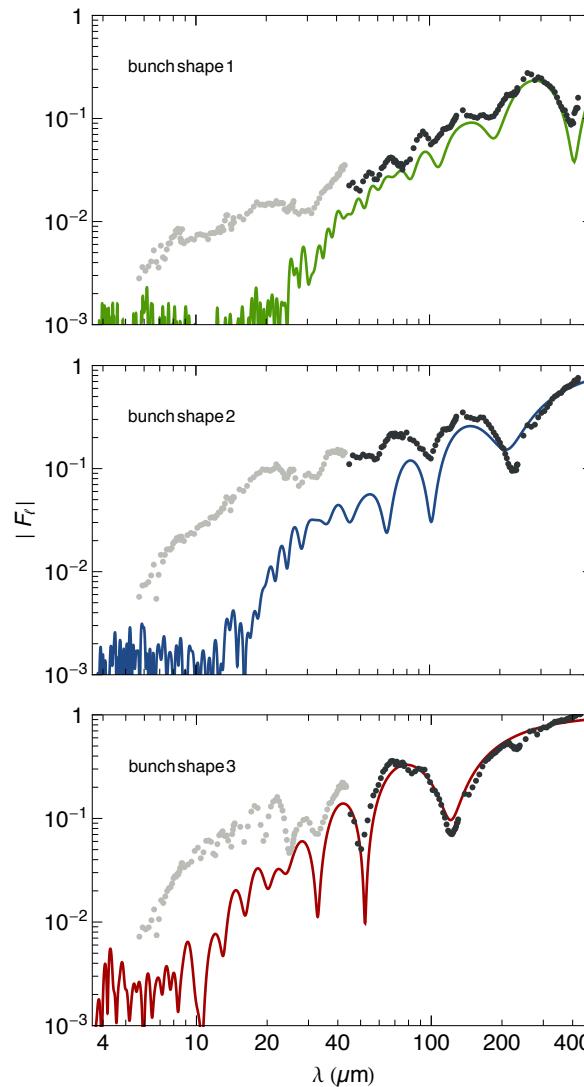
regularly used
as online
tool for monitoring
and compression
control

compare TDS and CRISP4 : |form factors|

Chapter



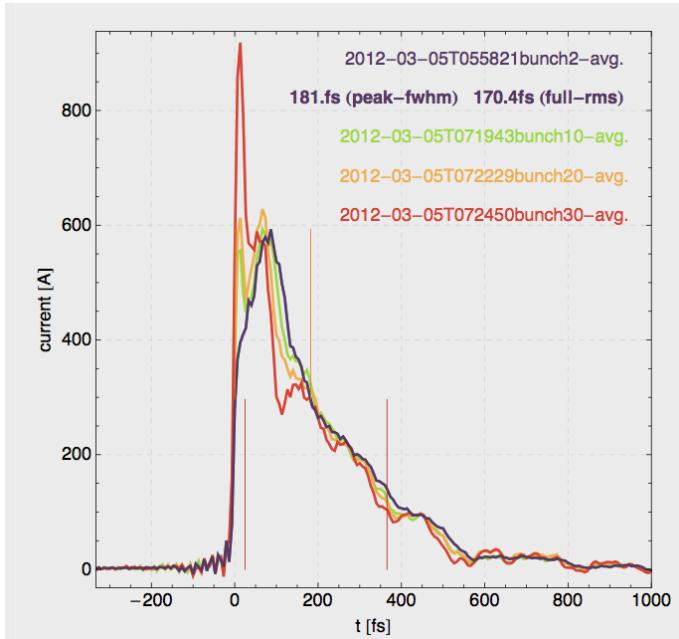
- very good agreement for long wavelengths
- TDS profiles show less short wavelength content
 - smoothed and averaged by tomography
 - finite resolution
 - TDS and CRISP4 are 60 m apart !



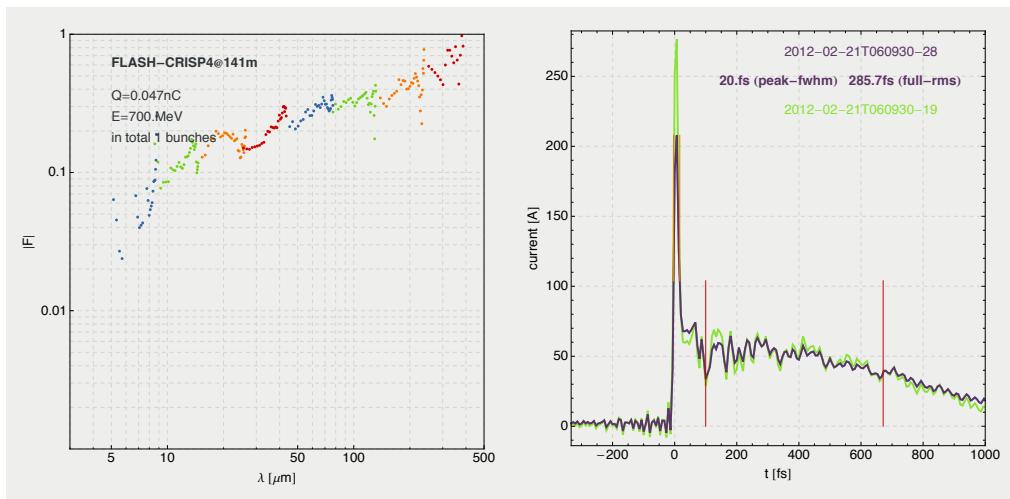
reconstructed shapes (KK-phase)

CRD

$Q = 0.137 \text{ nC}$
 $E = 500 \text{ MeV}$



- single bunch with 47 pC
- very sharp leading spike (**8.5 fs rms**) on long tail



reconstructed profiles along
bunch train

(using Krames-Kronig relation, not unique, „a possible bunch shape“)

outlook CR methods

CRD

shorter bunches, less charge... (few fs - pC bunches)

wavelength range $1 \mu\text{m} - 20 \mu\text{m}$

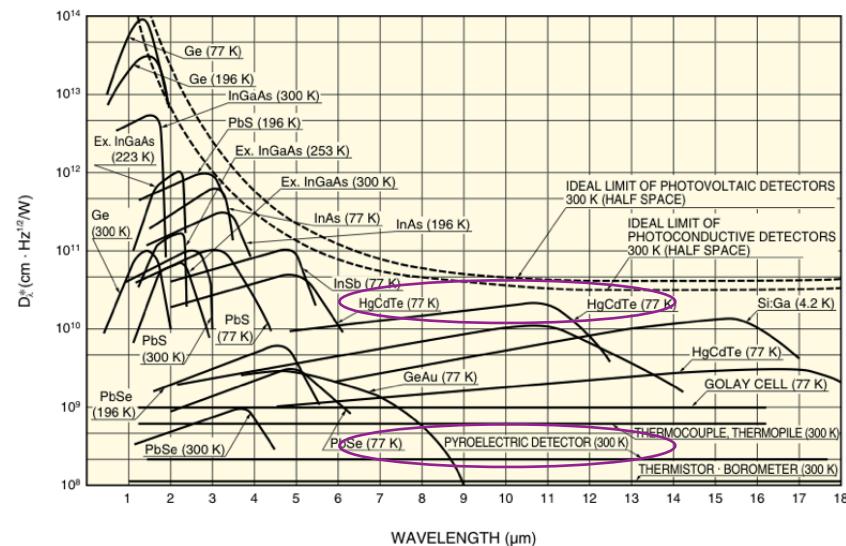
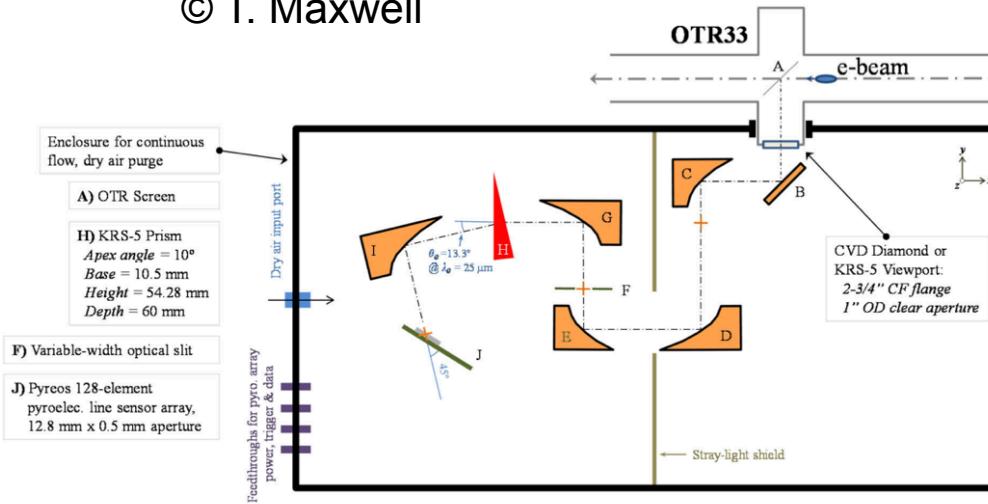
can be covered by prisms ! (KRS-5 or ZnSe)

more sensitive detectors (ex. **HgCdTe** arrays)



T. Maxwell
TUPA47

© T. Maxwell



Optical Replica Synthesizer

ORS

Nuclear Instruments and Methods in Physics Research A 539 (2005) 499–526

proposed by Saldin, Schneidmiller and Yurkov in 2004

„Operating range of proposed diagnostic technique nicely includes that of most ultrashort XFEL injector linacs, so it should be ideal for most everyday diagnostics as well as many more exotic ones“.

A simple method for the determination of the structure of ultrashort relativistic electron bunches

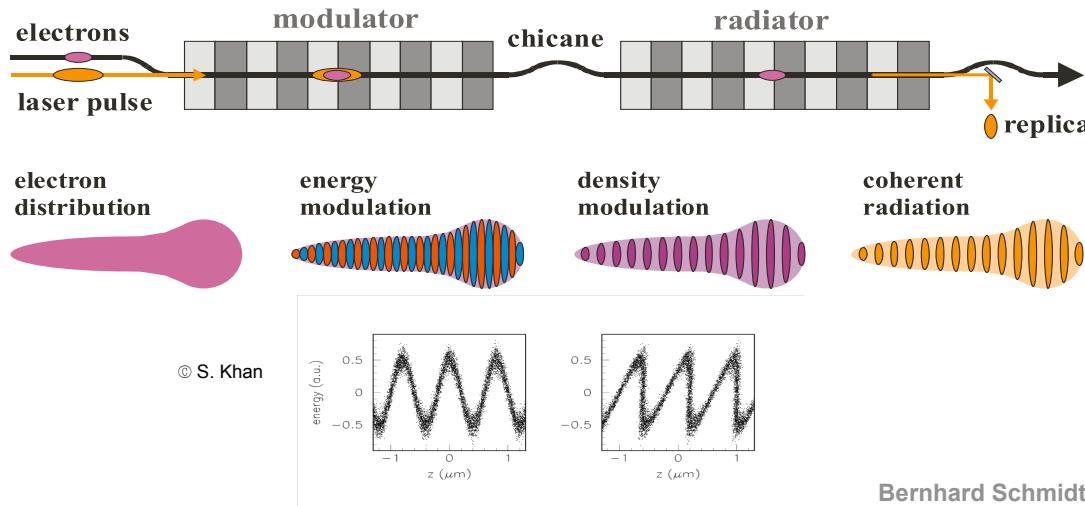
E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov*

Deutsches Elektronen-Synchrotron (DESY), Notkestrasse 85, Hamburg 22607, Germany

Received 3 August 2004; received in revised form 2 November 2004; accepted 2 November 2004

Available online 2 December 2004

- overlap beam with laser pulse in resonant undulator (*modulator*)
- generates energy modulation
- transform into density modulation by small chicane (R56)
- produce **coherent radiation** pulse at laser wavelength in second undulator
- measure temporal profile of „optical replica pulse“ with established optical methods (**FROG**,...)



ultimate resolution limit : phase slippage

$$\sigma_t \geq N_u \lambda / c$$

5 periods, 800nm \rightarrow 13 fs

beam constraints :

beam size in undulators small (diffraction limit)

$$\sigma_x \ll \sqrt{\lambda L_u / (2\pi)}$$

5 periods, 1 m $\rightarrow \sigma_x \ll 350 \mu\text{m}$

longitudinally „smooth“ beam (no microbunching)

technical constraints :

spatial and temporal overlap of laser and beam

time jitter laser to beam $<<$ laser pulse length

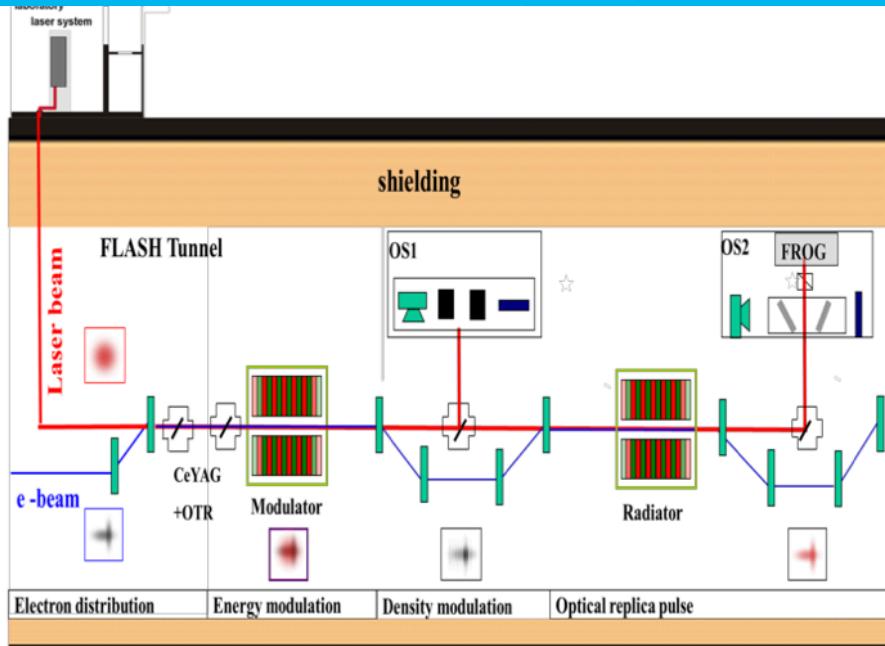
sufficient laser power to overcome intrinsic energy spread

sufficient radiated power for FROG operation

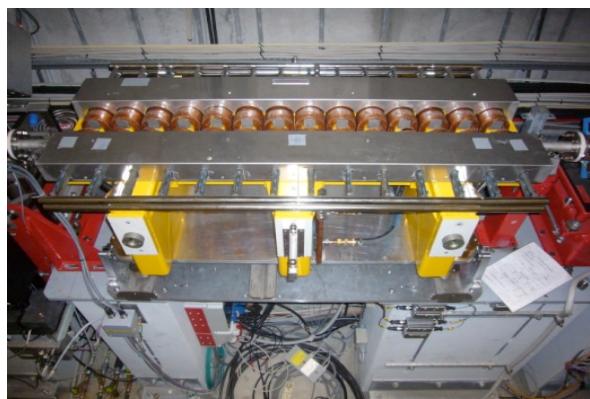
ORS installation at FLASH

ORS

G. Angelova et al, PRSTAB **11**, 070702 (2008)



E_{beam}	700 MeV
q	$0.5 - 1.2 nC$
σ_x	0.3 mm
$N_u \lambda_u$	$5 \cdot 0.2 m$
λ_L	800 nm
P_L	0.5 mJ
σ_L	400 fs
R_{56}	170 μm

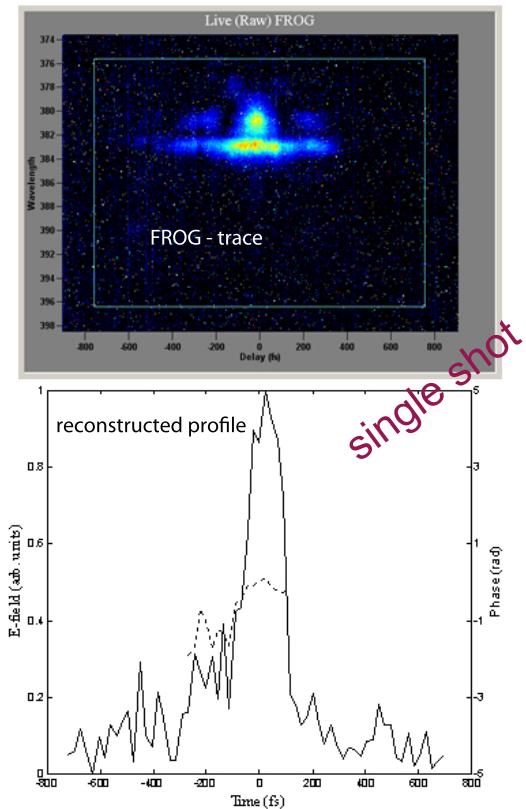


two orthogonal
undulators

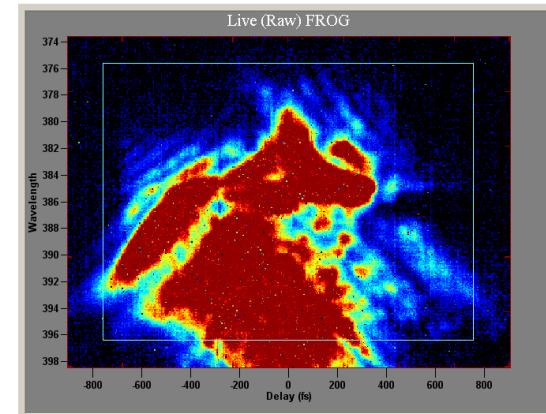


temporal and spatial overlap established

moderate signal for ~150 fs FWHM bunches



slightly more compression:



dominated by COTR from intrinsic microbunching ☹

avoid COTR by using **270 nm** seed beam (below COTR „cut-off“)

needs „transient grating (TG) FROG
with reflective optics

from T. Nagy and P. Simon, Optics Express 17, 8144 (2009)

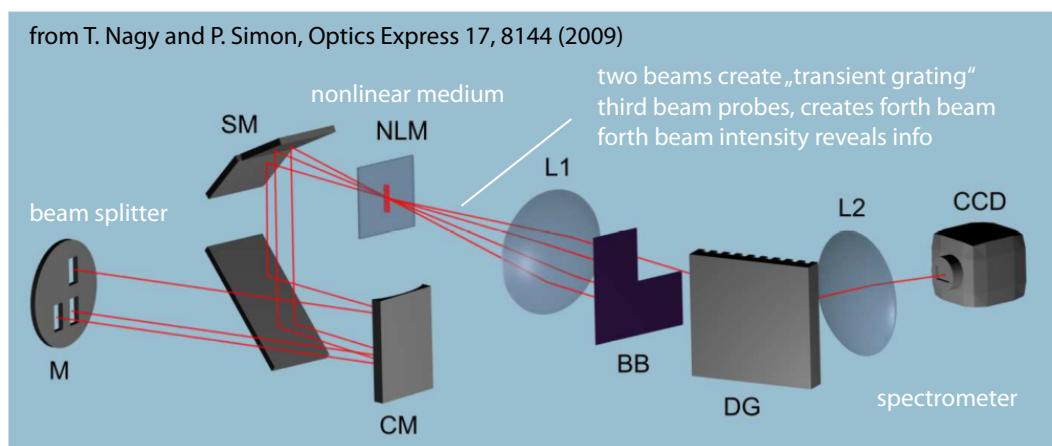


Fig. 2. The single-shot all-reflective TG FROG arrangement. (For the labels see text.)

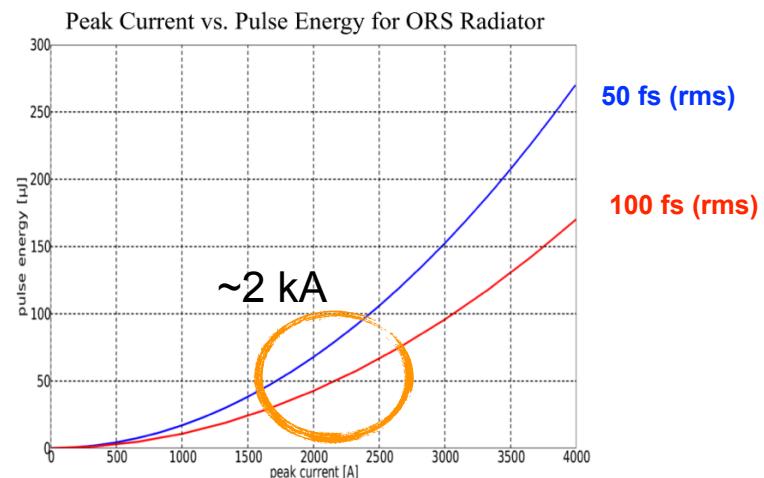
Proc. FEL-2012

OPTICAL REPLICA SYNTHESIZER TO BE RECOMMISSIONED WITH 270 NM SEED AT FLASH

Kirsten Hacker*, Robert Molo, Shaukat Khan, Technische Universitaet Dortmund, Germany

Peter Salen, Peter Van der Meulen, Stockholm University, Sweden

Gergana Angelova Hamberg, Volker Ziemann, Uppsala University, Sweden

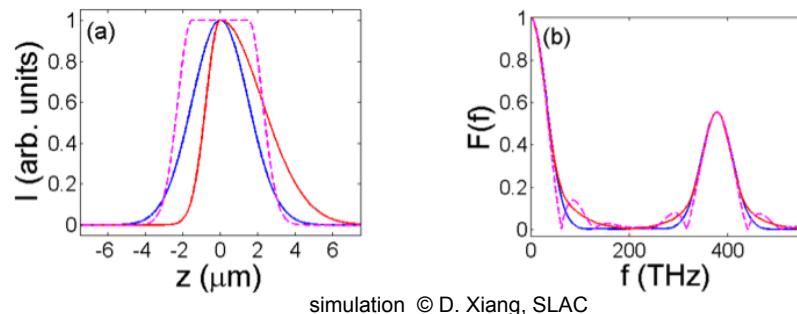


*interesting technique of non-vanishing complexity
requires considerable technical infrastructure
still in experimental phase*

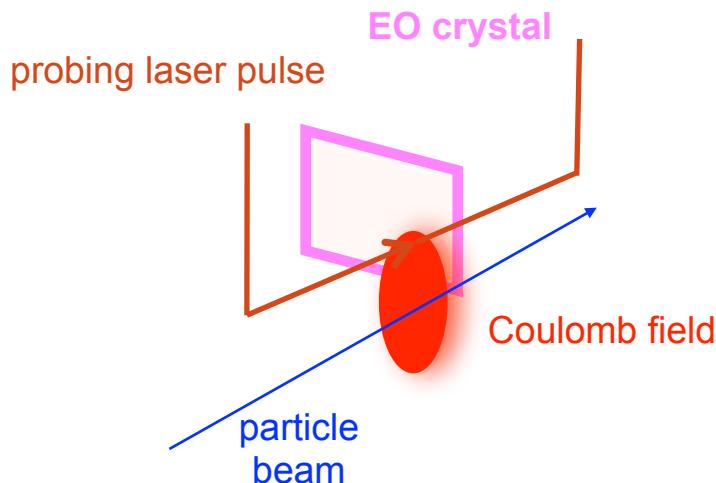
Dao Xiang : IPAC 2012 : the „simplified ORS“

no second undulator, no FROG \longrightarrow OTR screen + spectrometer

OTR spectrum reveals bunch shape
„up-converted“ THz spectrum
including phase problem !



Intra-beamline measurement of the bunch Coulomb field



- Field induced refractive index change
- Polarization-modulation of probing laser
- **Temporal structure** of Coulomb field i.e. (beam current)
→ impressed to ellipticity of optical pulse

$$\text{signal} \sim E_r(t) = \frac{I(t)}{2\pi b \epsilon_0 c}^{\gamma \gg 1}$$

$I(t)$ = current

VOLUME 85, NUMBER 16

PHYSICAL REVIEW LETTERS

16 OCTOBER 2000

Subpicosecond Electro-optic Measurement of Relativistic Electron Pulses

X. Yan,* A. M. MacLeod, and W. A. Gillespie

School of Science and Engineering, University of Abertay Dundee, Bell Street, Dundee DD1 1HG, United Kingdom

G. M. H. Knippels, D. Oepts, and A. F. G. van der Meer

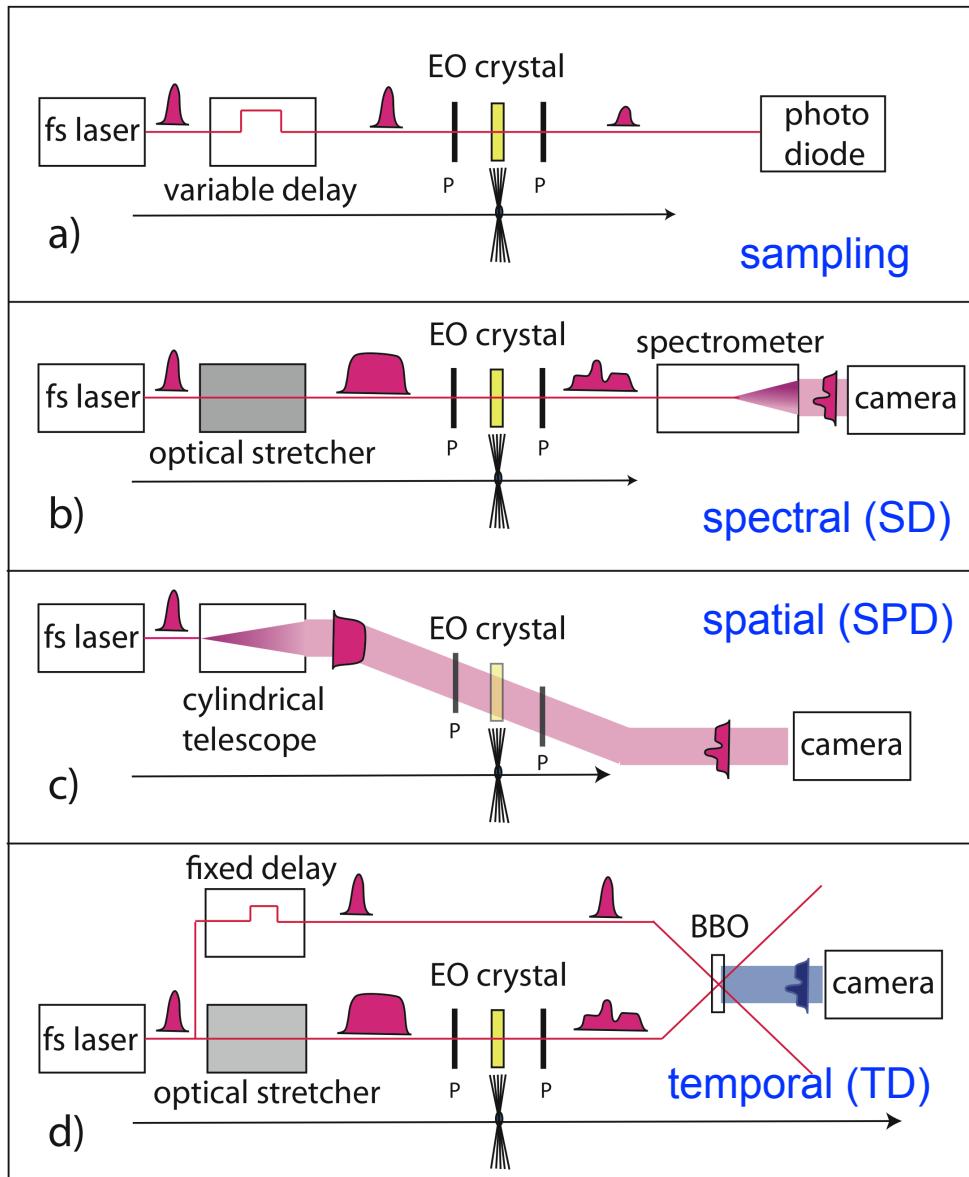
FOM-Instituut voor Plasmaphysica "Rijnhuizen," P.O. Box 1207, 3430 BE, Nieuwegein, The Netherlands

W. Seidel

Forschungszentrum Rossendorf, Institute for Nuclear and Hadron Physics, P.O. Box 51 01 19, 01314 Dresden, Germany
(Received 31 May 2000)

EO decoding schemes

EO



- ◆ classical
- ◆ needs very short laser pulse
- ◆ **time jitter << bunch length**
- ◆ multi-shot (scanning)

- ◆ linearly chirped laser pulse
- ◆ **converts time → wavelength**
- ◆ technically simple
- ◆ systematic distortions and bias

- ◆ **cross-correlation technique in vacuum**
- ◆ detection system (imaging) close to beam
- ◆ bias free up to „crystal limits“

- ◆ **probe + sample pulse**
- ◆ SHG needs considerable power
- ◆ more demanding
- ◆ bias free up to „crystal limits“

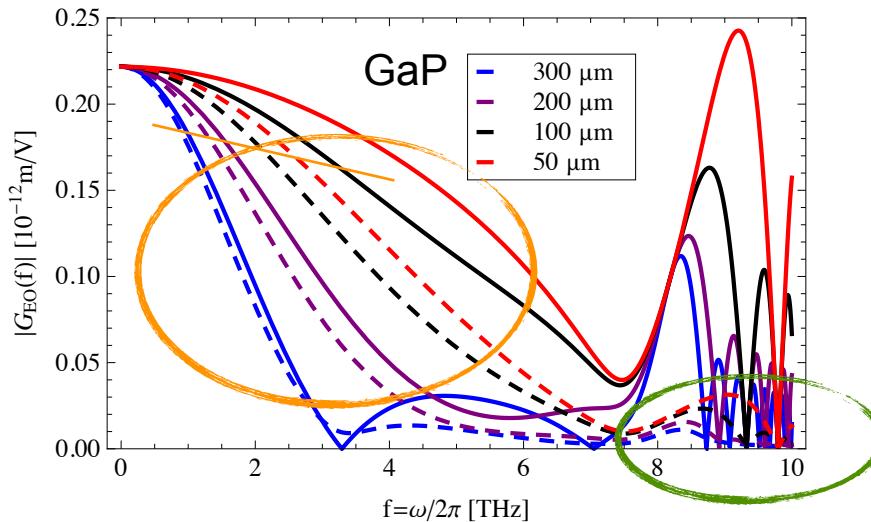
Electro optic : resolution limits

EO

Limits from anorganic EO material : phonon resonances & phase slipping

avoid „slipping“ by thin crystals
less signal

the EO „response function“, $\lambda=800$ nm



$\sigma_b \geq 50$ fs - GaP

$\sigma_b \geq 100$ fs - ZnTe

short bunches → high frequencies
excite phonon resonances
complex response, ringing...

status spectral decoding EO-SD

EO

Problem :

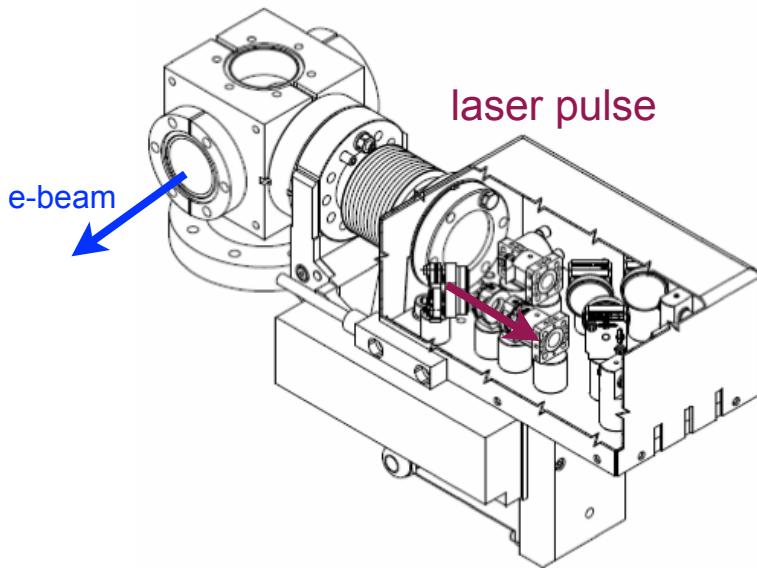
frequency mixing of bunch field an laser **distorts the „linear chirp“** (the „time scale“).

→ SD restricted to „long“ bunches. 100 μm GaP : $\sigma_b \geq 350$ fs

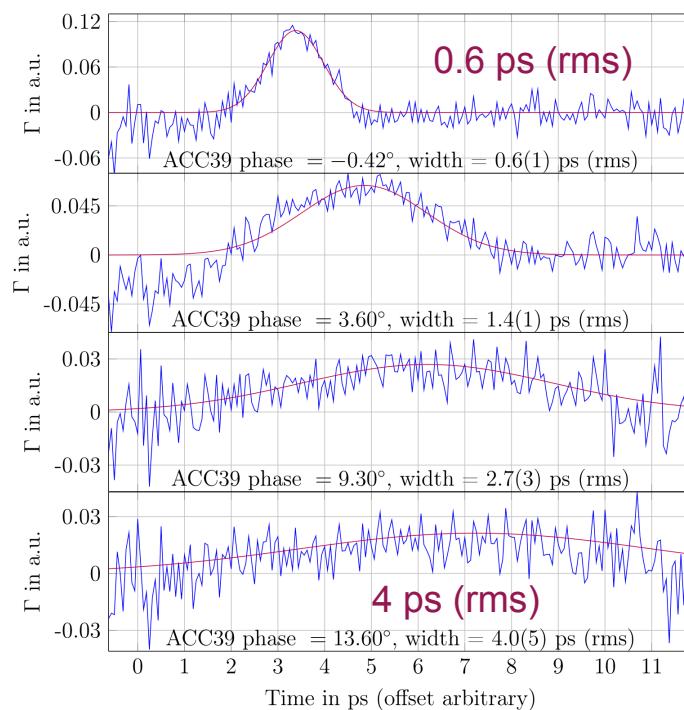
But : technically less demanding, robust, compact

Example : online monitor developed at PSI & DESY (Proc. DIPAC 09, B.Steffen,V. Schlott, F. Müller)

- Yb fibre laser (1030 nm) avoids delicate TiSa
- almost perfect phase matching, thick crystals allowed
- read out by commercial InGaAs spectrometer (fibre coupled)



drawback: finding the temporal overlap is (still) cumbersome



FLASH after first compressor

Spatial Decoding

EO

first demonstrated at SLAC, 2005

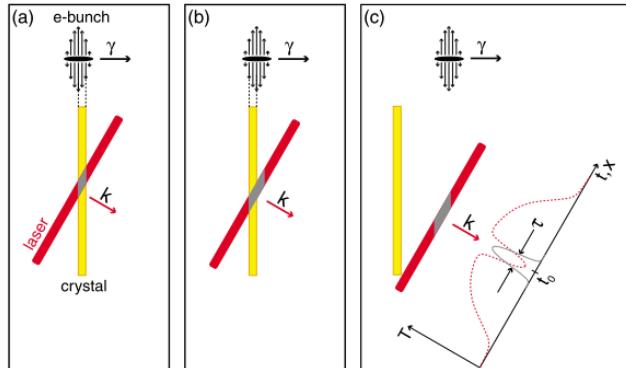
PRL 94, 114801 (2005)

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week ending
25 MARCH 2005

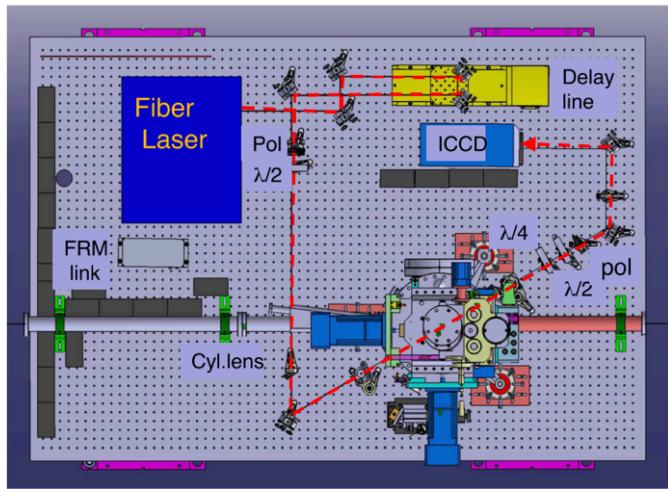
Clocking Femtosecond X Rays

A. L. Cavalieri,¹ D. M. Fritz,¹ S. H. Lee,¹ P. H. Bucksbaum,¹ D. A. Reis,¹ J. Rudati,² D. M. Mills,² P. H. Fuoss,³ G. B. Stephenson,³ C. C. Kao,⁴ D. P. Siddons,⁴ D. P. Lowney,⁵ A. G. MacPhee,⁵ D. Weinstein,⁵ R. W. Falcone,⁵ R. Pahl,⁶ J. Als-Nielsen,⁷ C. Blome,⁸ S. Düsterer,⁸ R. Ischebeck,⁸ H. Schlarb,⁸ H. Schulte-Schrepping,⁸ Th. Tschentscher,⁸ J. Schneider,⁸ O. Hignette,⁹ F. Sette,⁹ K. Sokolowski-Tinten,¹⁰ H. N. Chapman,¹¹ R. W. Lee,¹¹ T. N. Hansen,¹² O. Synnergren,¹² J. Larsson,¹² S. Techert,¹³ J. Sheppard,¹⁴ J. S. Wark,¹⁴ M. Bergh,¹⁵ C. Caleman,¹⁵ G. Hultd,¹⁵ D. van der Spoel,¹⁵ N. Timneanu,¹⁵ J. Hajdu,¹⁵ R. A. Akre,¹⁶ E. Bong,¹⁶ P. Emma,¹⁶ P. Krejcik,¹⁶ J. Arthur,¹⁷ S. Brennan,¹⁷ K. J. Gaffney,¹⁷ A. M. Lindenberg,¹⁷ K. Luening,¹⁷ and J. B. Hastings¹⁷

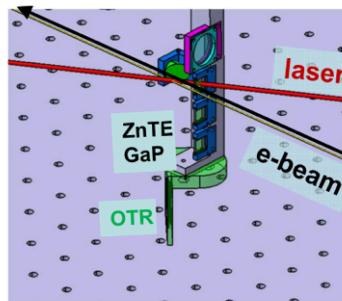


M. Veronese, BIW12

TUPA43



FIRST EOS measurements
Spatial encoding scheme
Fiber Laser : 780nm -FW 100fs
ZnTe, GaP, OTR, YAG



recent example

single shot technique, resolution not limited by frequency mixing
ideal for arrival time monitoring !

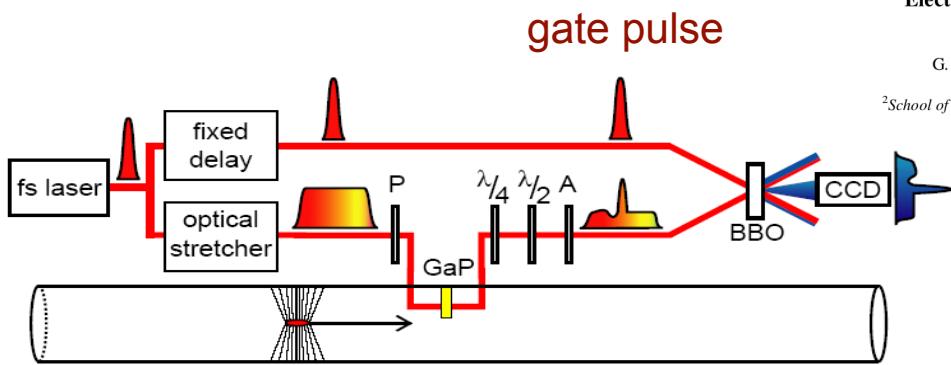
Temporal decoding

EO

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10 SEPTEMBER 2004



Electro-Optic Technique with Improved Time Resolution for Real-Time, Nondestructive, Single-Shot Measurements of Femtosecond Electron Bunch Profiles

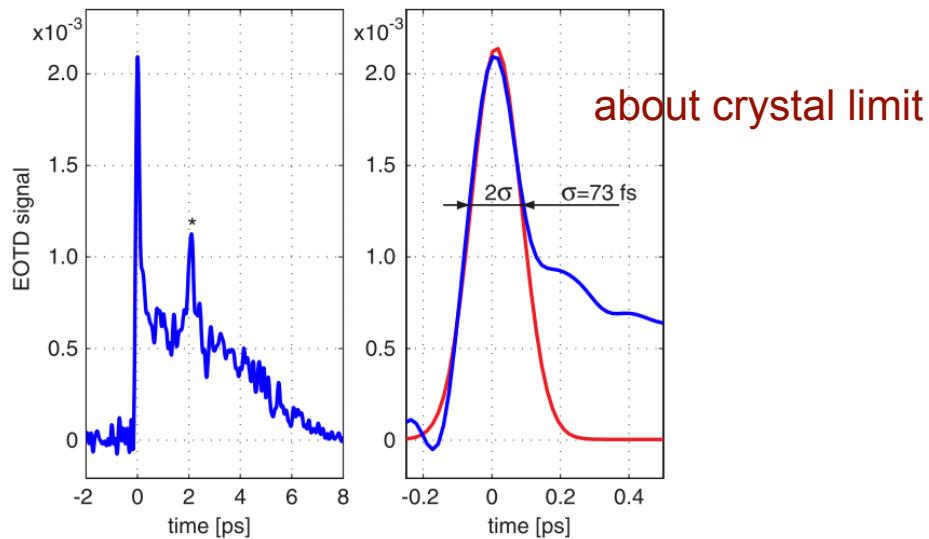
G. Berden,¹ S. P. Jamison,^{2,*} A. M. MacLeod,² W. A. Gillespie,² B. Redlich,¹ and A. F. G. van der Meer¹

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(Received 22 April 2004; published 9 September 2004)

cross correlation in
BBO crystal



needs $> \mu\text{J}$ laser power (2.order process)

complex optics

rather experiment but monitor

Extending electro-optic detection to ultrashort electron beams

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Plasma Physics Division, Naval Research Laboratory, Washington, D.C. 20375, USA
 (Received 29 September 2011; revised manuscript received 23 February 2012; published 3 May 2012)

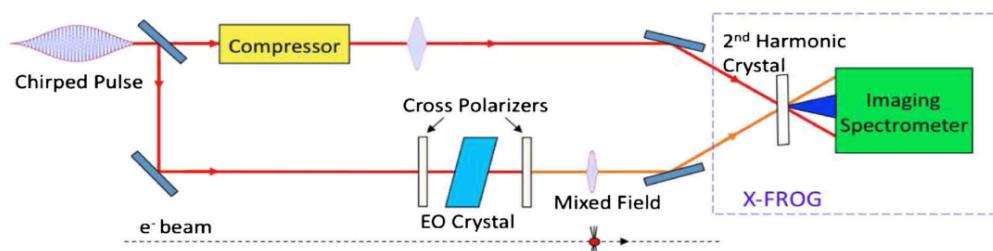
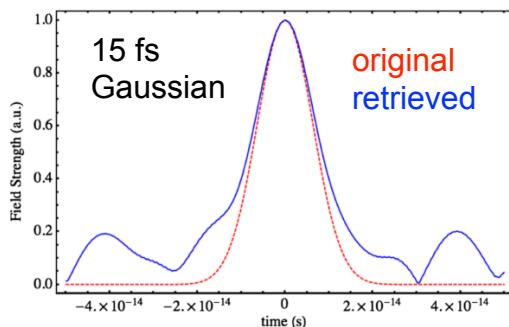


FIG. 2. BMX-FROG setup used to measure ultrashort charged particle bunches.

add spectrometer to EO-TD set-up



caveat:

- needs very precise knowledge of crystal response
- inversion of complex profiles ?

overcome problems
with resonances

simulated FROG images

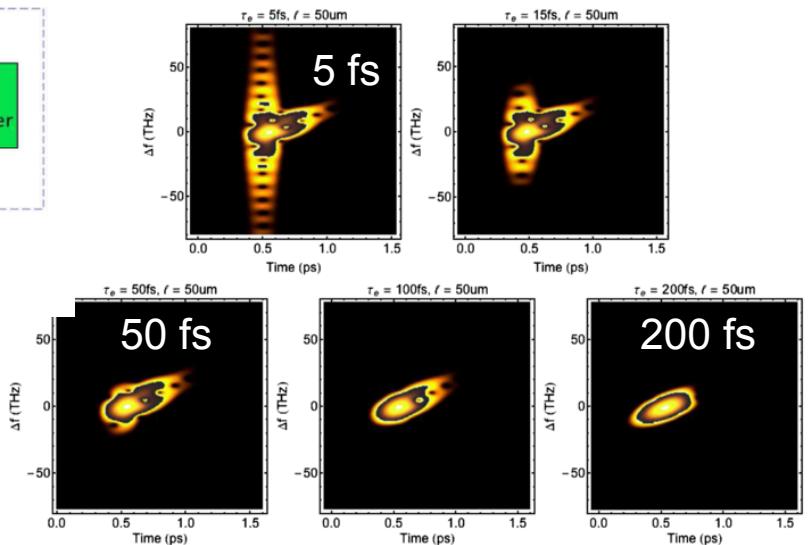


FIG. 5. Simulation results for various bunch lengths measured using a $50 \mu\text{m}$ crystal with a chirped probe. Each image is plotted using its own normalized false color scale.

EO : frequency up-conversion

EO

WEPC80

Proceedings of FEL2009, Liverpool, UK

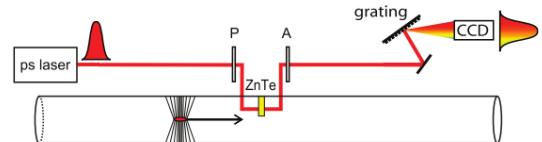
LONGITUDINAL ELECTRON BEAM DIAGNOSTICS VIA UPCONVERSION OF THZ TO VISIBLE RADIATION

G. Berden, A.F.G. van der Meer, FELIX / FOM Institute 'Rijnhuizen', Nieuwegein, NL

S.P. Jamison, ASTEC, Daresbury Laboratory, STFC, UK

A.M. MacLeod, Abertay University, Dundee, UK

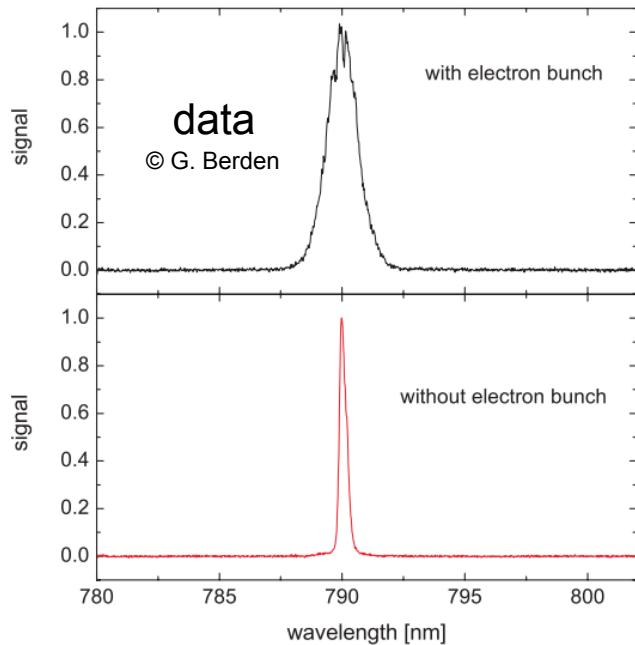
W.A. Gillespie, P.J. Phillips, Dundee University, Dundee, UK



most simple set-up
needs narrow band (long) laser pulse

spectral broadening („side bands“)
by THz spectrum of bunch

no phase information (yet) ! (as with CTR spectra..)



a really ambitious project

EO

WEPPH053

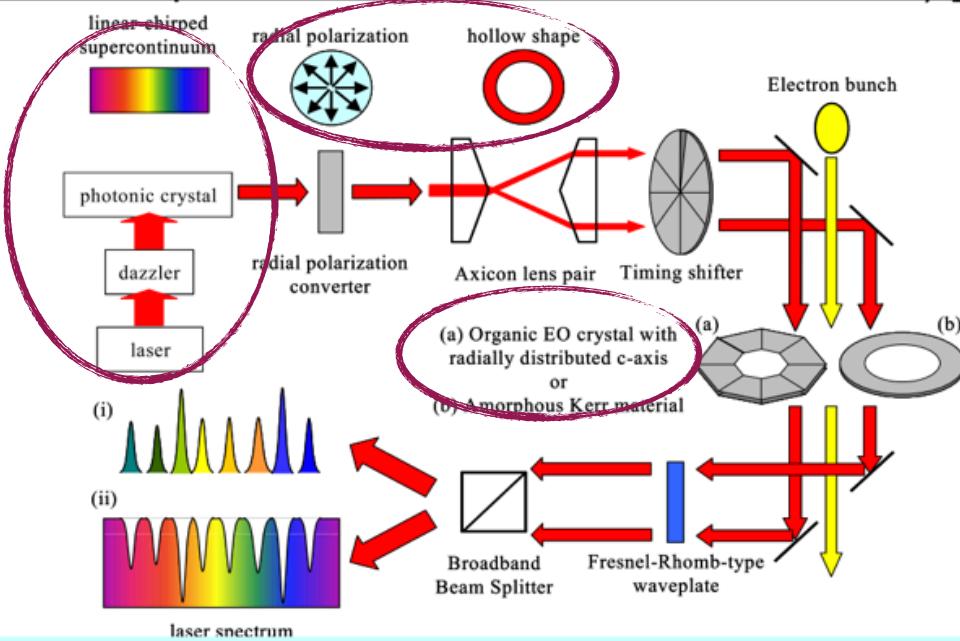
Proceedings of FEL 2007, Novosibirsk, Russia

NON-DESTRUCTIVE SINGLE-SHOT 3-D ELECTRON BUNCH MONITOR WITH FEMTOSECOND-TIMING ALL-OPTICAL SYSTEM FOR PUMP & PROBE EXPERIMENTS

H. Tomizawa, H. Hanaki, Accelerator Division, Japan Synchrotron Radiation Research Institute (JASRI/SPring-8), Kouto, Sayo-cho, Sayo-gun, Hyogo 679-5198, Japan
T. Ishikawa, The Institute of Physical and Chemical Research (RIKEN Harima/SPring-8), Kouto, Sayo-cho, Sayo-gun, Hyogo 679-5148, Japan

talk by Y. Okayasu
MOCC03

3D bunch shape monitor (One element of 3D-BCDM) [1]



Supercontinuum generation: Photonic crystal fiber

Supercontinuum propagation: Spectrum modulation by DAZZLER + Developments of optics

radial polarization: Liquid crystal

Hollow shape: Axicon mirror pair

codes time & radial information into spectrum

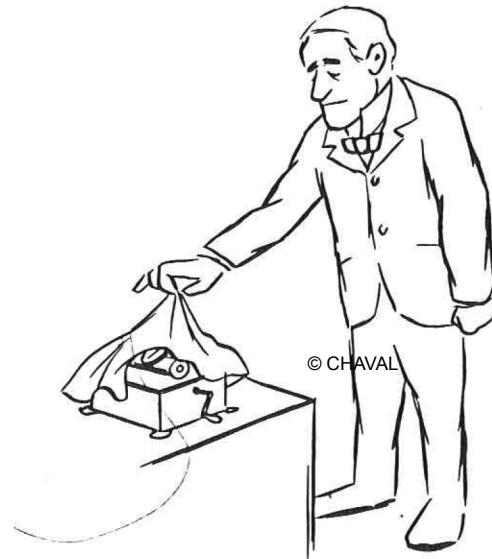
several new and challenging concepts !

under development

Summary

- progress in TDS technology towards ~ 1 fs regime
- coherent radiation methods made it to online monitors, perspectives for shorter bunches
- electro-optic methods : more experiment than monitors, interesting approaches ahead
- ORS : proof of feasibility still missing, complex technology
- interesting new ideas and concepts waiting to be tested

Thank you and all contributors !



there is always room for the discovery of wonderful new techniques...

additional stuff

Chapter



fs resolution achieved: „zero phasing“

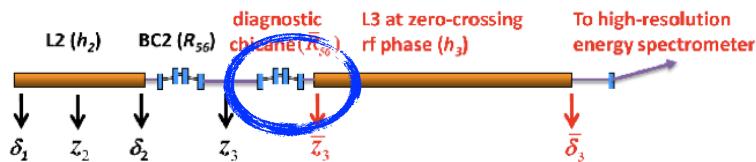
Proceedings of 2011 Particle Accelerator Conference, New York, NY, USA

THP183

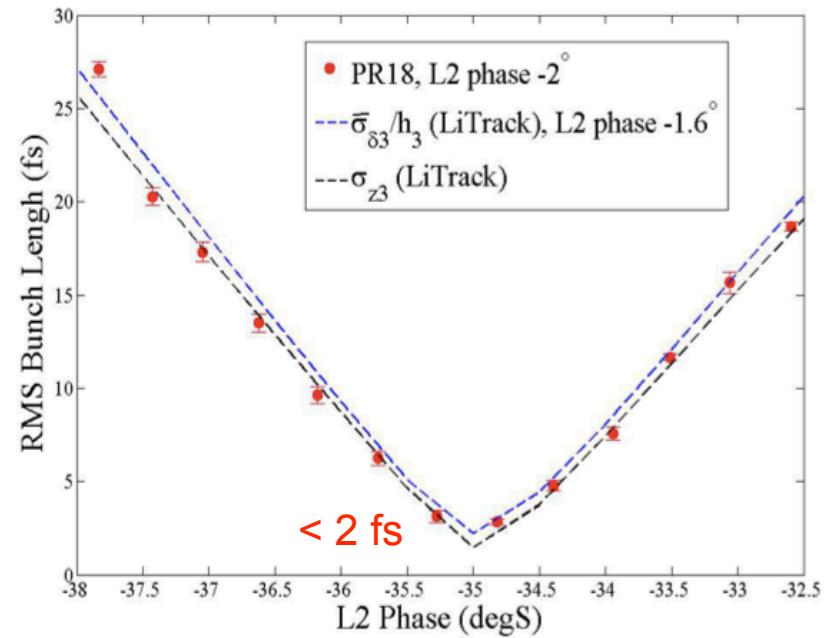
MEASUREMENT OF FEMTOSECOND LCLS BUNCHES USING THE SLAC A-LINE SPECTROMETER *

Z. Huang, A. Baker, C. Behrens[†], M. Boyes, J. Craft, F.-J. Decker,
Y. Ding, P. Emma, J. Frisch, R. Iverson, J. Lipari, H. Loos, D. Walz
SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

dedicated experiment at LCLS
running linac at „zero phase“
chirp maps time → energy
high resolution(10^{-4}) „A-line“ spectrometer



compensates
intrinsic energy spread



Back to time domain : the phase problem

CRD

$$F(\lambda) = \int_{-\infty}^{\infty} S(z) e^{-2\pi iz/\lambda} dz$$

(Lay, Happek, Sievers 1994)
complex function, $S(z)$ is real

Kramers-Kronig relation: connects real and imaginary part (for a certain class of S !)

$$\phi_m(\lambda_0) = -\frac{2}{\pi} \lambda_0 \int_0^{\infty} \frac{\log(|F(\lambda_0)| / |F(\lambda)|)}{(\lambda_0^2 - \lambda^2)} d\lambda$$

needs $|F(\lambda)|$ over a wide range ..
but neither „0“ nor „infinity“..
extrapolation needed (long wavelengths)

$$S_m(z) = \frac{2}{c\lambda^2} \int_0^{\infty} |F(\lambda)| \cos\left(\frac{2\pi}{\lambda} + \phi_m(\lambda)\right) d\lambda$$

measured values

„minimal“ or „canonical“ bunch profile

EOSD: early days

EO

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25 MARCH 2002

Single-Shot Electron-Beam Bunch Length Measurements

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(Received 10 September 2001; published 6 March 2002)

