IR optics

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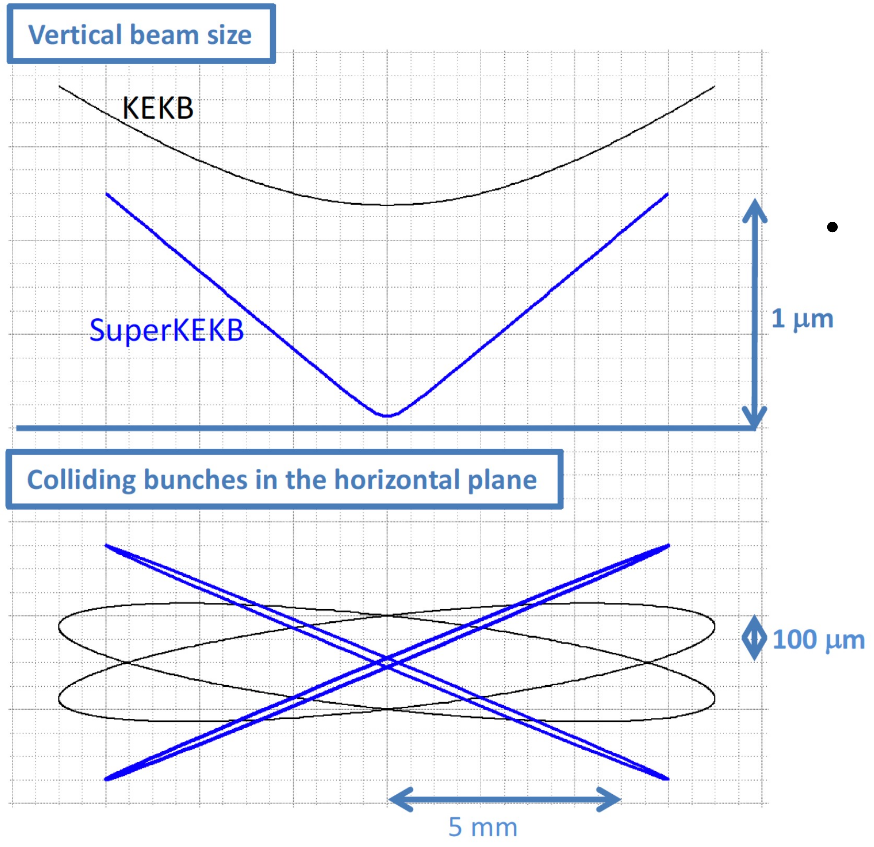
2025.6.16

B2GM

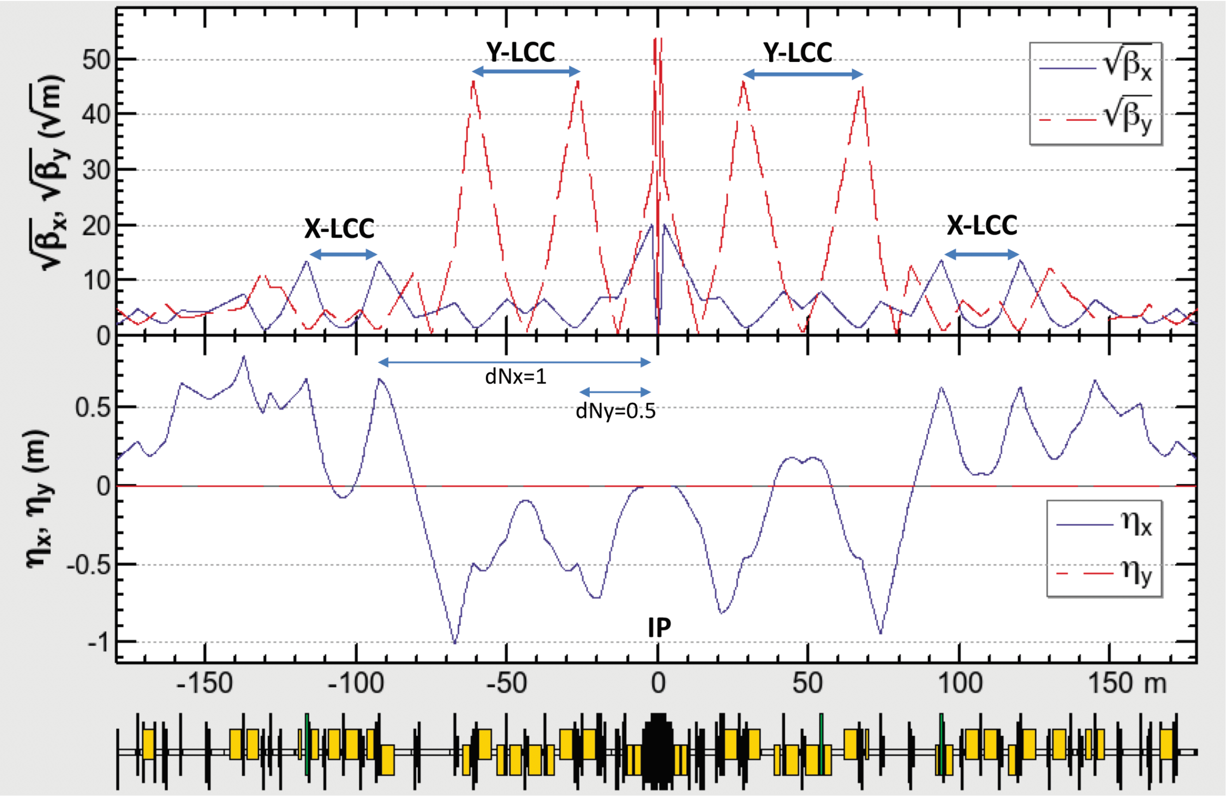
IR optics



- The design strategy for SuperKEKB is based on the nanobeam scheme[1], in which bunches with small σ_x^* collide at a large crossing angle. The longitudinal size of overlap between colliding bunches decreases ~ 1/20 of the bunch length. To achieve this condition, σ_x^* should be sufficiently small, which means both small β_x^* and small horizontal emittance are required.

[1] "SuperB Conceptual Design Report", INFN/AE-07/2, SLAC-R-856, LAL 07-15, March 2007

IR optics

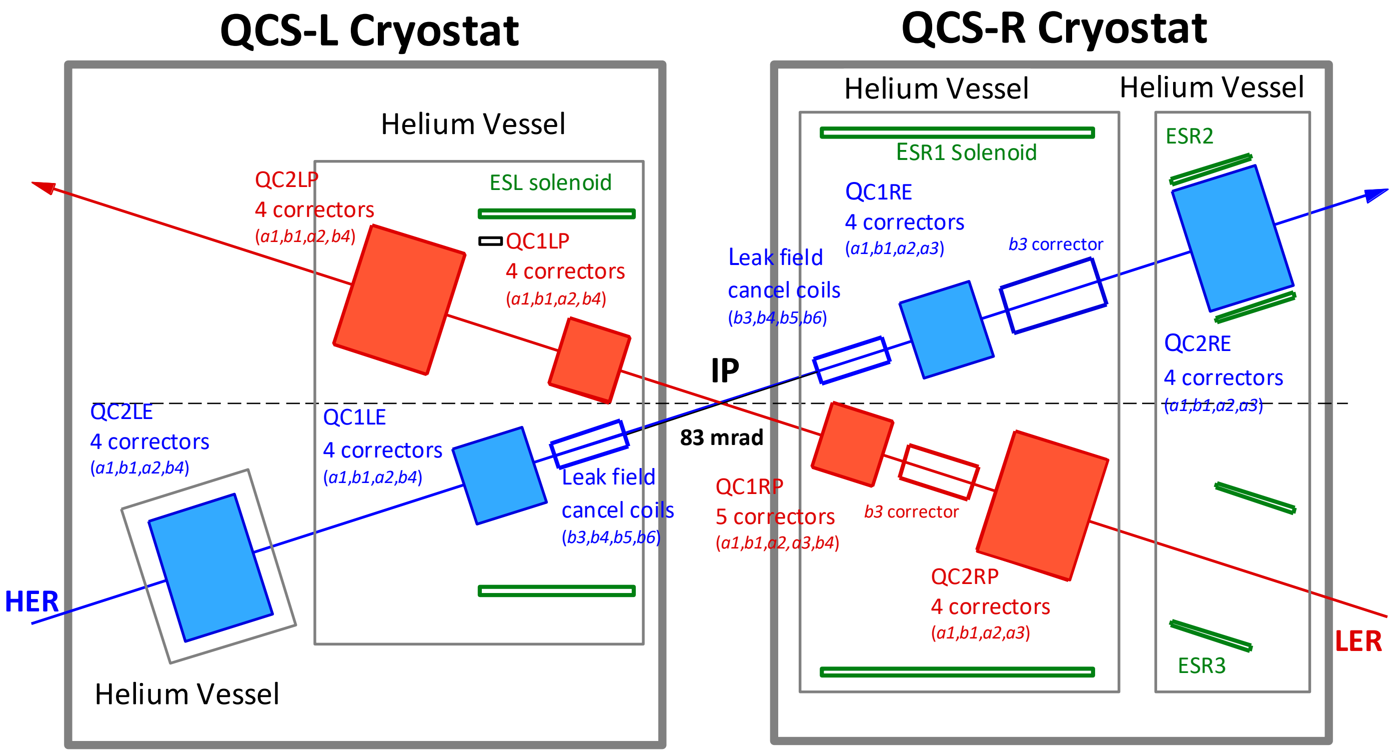


$$-I' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ m_{12} & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & m_{34} & -1 \end{pmatrix}$$

- To correct large chromaticity arising from small β^*, local chromaticity correction (LCC) sections for both the vertical and horizontal planes. A pair of identical sextupole magnets, connected by the pseudo -I transformation, are placed in each LCC.

QCS system

- The QCS system is equipped with various corrector magnets: vertical and horizontal dipole (a1 and b1), skew quadrupole (a2), skew and normal sextupole (a3 and b3), normal octupole, decapole, and dodecapole (b4, b5, and b6) magnets.



4 SC main quadrupole magnets: 1 collared magnet, 3 yoked magnets
16 SC correctors: a1, b1, a2, b4
4 SC leak field cancel magnets: b3, b4, b5, b6
1 compensation solenoid

4 SC main quadrupole magnets: 1 collared magnet, 3 yoked magnets
19 SC correctors: a1, b1, a2, a3, b3, b4
4 SC leak field cancel magnets: b3, b4, b5, b6
3 compensation solenoid

N. Ohuchi, et al. "Design of the superconducting magnet system for the SuperKEKB interaction region", in Proceedings of the NA-PAC'13, Pasadena, CA USA, pp.759– 761

IR optics

- There are many interesting topics on the IR optics, but this talk mainly discuss on some of the basics of beam optics such as transfer matrix, Twiss parameters, betatron oscillation, etc.

Motion of beam particles

- Optics design for SuperKEKB is done using SAD.
- The lecture materials “Basics of SAD” by Oide san at the SAD School are very useful for an introduction to beam optics. I will translate and use some of them in today's talk.

https://superkekb.kek.jp/category/8/

2011年6月

- 6月29日 Yukiyoshi Ohnishi, "SAD利用（２）"
- 6月22日 Katsunobu Oide, "SADのビーム物理"
- 6月15日 Yukiyoshi Ohnishi, "SAD利用（１）"
- 6月08日 Akio Morita, "SADの文法"
- 6月01日 Akio Morita, "SADの文法"

2011年5月

- 5月25日 Katsunobu Oide, "SADのビーム物理"
- 5月18日 Susumu Kamada, "電子貯蔵リングのビームエミッタンス"
- 5月11日 Susumu Kamada, "SAD演習：FODOセルリングのマッチング（続）"

2011年4月

- 4月28日 Etienne Forest, "ビーム力学の講義（第2回)"
- 4月27日 Susumu Kamada, "SAD演習：FODOセルリングのマッチング"
- 4月22日 Etienne Forest, "ビーム力学の講義（第1回)"
- 4月20日 Katsunobu Oide, "SADのビーム物理"
- 4月13日 Susumu Kamada, "SAD：簡単な使用例"

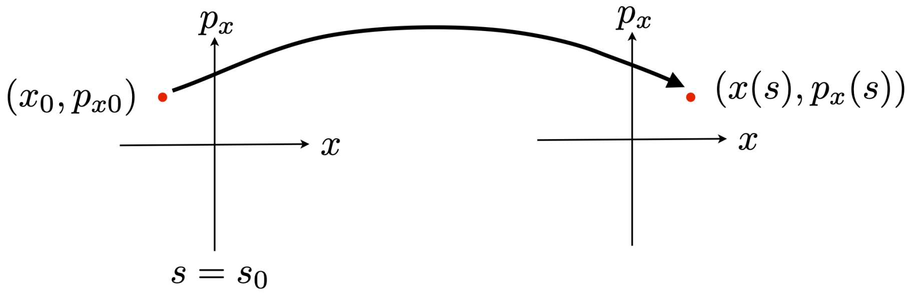
Motion of beam particles

- The equations of motion for beam particles are expressed by Hamilton's equations of motion.

$$x' = \frac{dx}{ds} = \frac{\partial H}{\partial p_x} \qquad\qquad p_x' = \frac{dp_x}{ds} = -\frac{\partial H}{\partial x}$$

- The independent variable s is the length along a certain reference orbit. The reference orbit need not be an actual path of the beam particles.

- When there are given the position x_0 and momentum p_{x0} of a beam particle at a location s_0 on the beamline, the position $x(s)$ and momentum $p_x(s)$ are determined at any given location s.



Excerpted and translated from SAD_20110420_Oide.pdf
K. Oide, "SADのビーム物理", 2011年4月20日
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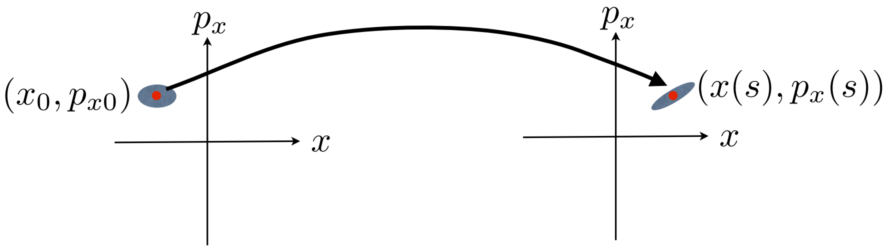
Transfer matrix

- $x(s)$ and $p_x(s)$ can be regarded as functions of x_0 and p_{x0} .

$$x(x_0, p_{x0}; s), \quad p_x(x_0, p_{x0}; s)$$



- Expand $x(s)$ and $p_x(s)$ around x_0 and p_{x0} to find the first order coefficients. These first-order coefficients are called the transfer matrix.

$$M \equiv \begin{pmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial p_{x0}} \\ \frac{\partial p_x}{\partial x_0} & \frac{\partial p_x}{\partial p_{x0}} \end{pmatrix} = \begin{pmatrix} \frac{\partial x(x_0,p_{x0};s)}{\partial x_0} & \frac{\partial x(x_0,p_{x0};s)}{\partial p_{x0}} \\ \frac{\partial p_x(x_0,p_{x0};s)}{\partial x_0} & \frac{\partial p_x(x_0,p_{x0};s)}{\partial p_{x0}} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

- The motion of beam particles in accelerators is controlled by the transfer matrix.

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Transfer matrix

- The determinant of the transfer matrix det $M(s)$ is 1 at $s = s_0$. Differentiating det $M(s)$ by s yields zero, so det $M(s)$ is invariant therefore det $M(s)$ is always 1.

$$\det M(s_0) = \frac{\partial x_0}{\partial x_0}\frac{\partial p_{x0}}{\partial p_{x0}} - \frac{\partial x_0}{\partial p_{x0}}\frac{\partial p_{x0}}{\partial x_0} = 1 - 0 = 1$$

$$\begin{aligned}
(\det M(s))' &= \left(\frac{\partial x}{\partial x_0}\frac{\partial p_x}{\partial p_{x0}} - \frac{\partial x}{\partial p_{x0}}\frac{\partial p_x}{\partial x_0}\right)' \\
&= \frac{\partial x'}{\partial x_0}\frac{\partial p_x}{\partial p_{x0}} + \frac{\partial x}{\partial x_0}\frac{\partial p_x'}{\partial p_{x0}} - \frac{\partial x'}{\partial p_{x0}}\frac{\partial p_x}{\partial x_0} - \frac{\partial x}{\partial p_{x0}}\frac{\partial p_x'}{\partial x_0} \\
&= \left(\frac{\partial}{\partial x_0}\left(\frac{\partial H}{\partial p_x}\right)\right)\frac{\partial p_x}{\partial p_{x0}} - \frac{\partial x}{\partial x_0}\left(\frac{\partial}{\partial p_{x0}}\left(\frac{\partial H}{\partial x}\right)\right) - \cdots \\
&= \frac{\partial}{\partial x_0}\left(\frac{\partial H}{\partial p_{x0}}\right) - \frac{\partial}{\partial p_{x0}}\left(\frac{\partial H}{\partial x_0}\right) \\
&= 0
\end{aligned}$$

$$x' = \frac{\partial H}{\partial p_x}$$

$$p_x' = -\frac{\partial H}{\partial x}$$

$$\det M(s) = \frac{\partial x}{\partial x_0}\frac{\partial p_x}{\partial p_{x0}} - \frac{\partial x}{\partial p_{x0}}\frac{\partial p_x}{\partial x_0} = 1$$

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Transfer matrix

- In general, a one-dimensional transfer matrix can be expressed as follows,

$$M = \left[\begin{pmatrix} 1 & 0 \\ \alpha & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{\beta} & 0 \\ 0 & \sqrt{\beta} \end{pmatrix}\right]^{-1}\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ \alpha_0 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{\beta_0} & 0 \\ 0 & \sqrt{\beta_0} \end{pmatrix}\right]$$

$(u, p_u) \to (x, p_x)$
normal physical

rotation
in the normal coordinate

$(x, p_x) \to (u, p_u)$
physical normal

$$(u, p_u) = \left(\frac{x}{\sqrt{\beta}},\ p_x\sqrt{\beta} + x\frac{\alpha}{\sqrt{\beta}}\right), \qquad (x, p_x) = \left(\sqrt{\beta}u, \frac{p_u - \alpha u}{\sqrt{\beta}}\right)$$

- The beam can always be circular in the normal coordinate by providing two parameters α and β, Twiss parameters, at the entrance and exit, respectively.

- In the normal coordinate, the transfer matrix is simply a rotation in phase space. Thus, the sum of squares of the amplitudes is constant. 2J : Courant-Snyder Invariant

$$2J_u \equiv u^2 + p_u^2 = x^2/\beta + (p_x + x\alpha/\beta)^2\beta = \frac{1+\alpha^2}{\beta}x^2 + 2\alpha x p_x + \beta p_x^2 = \text{const.}$$

2J : Courant-Snyder Invariant

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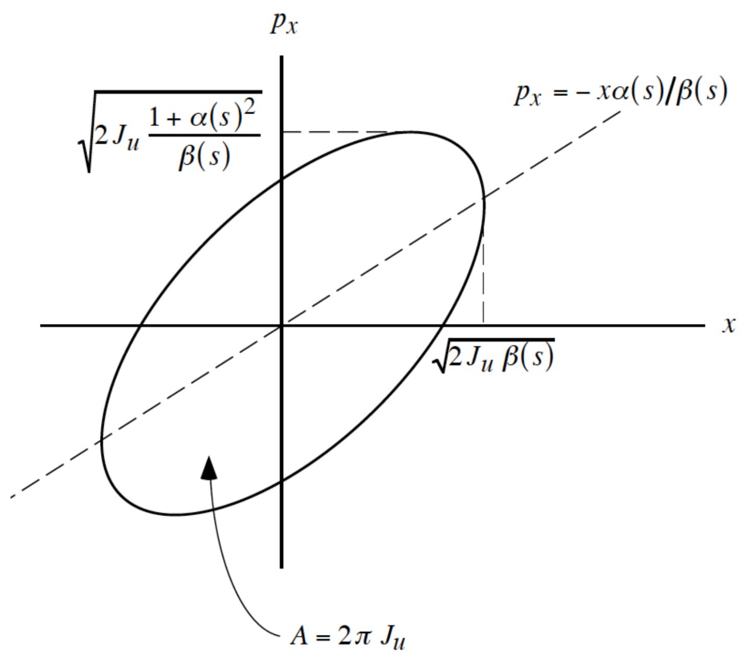
Betatron oscillation

- The betatron oscillation in the normal coordinates can be expressed simply as follows,

$$u(s) = \sqrt{2J_u}\sin(\phi(s) + \phi_0)$$
$$p_u(s) = \sqrt{2J_u}\cos(\phi(s) + \phi_0)$$

- In the physical phase space, particles with identical J_u and different phases are arranged elliptically.

$$(x, p_x) = \left(\sqrt{\beta}u, \frac{p_u - \alpha u}{\sqrt{\beta}}\right)$$



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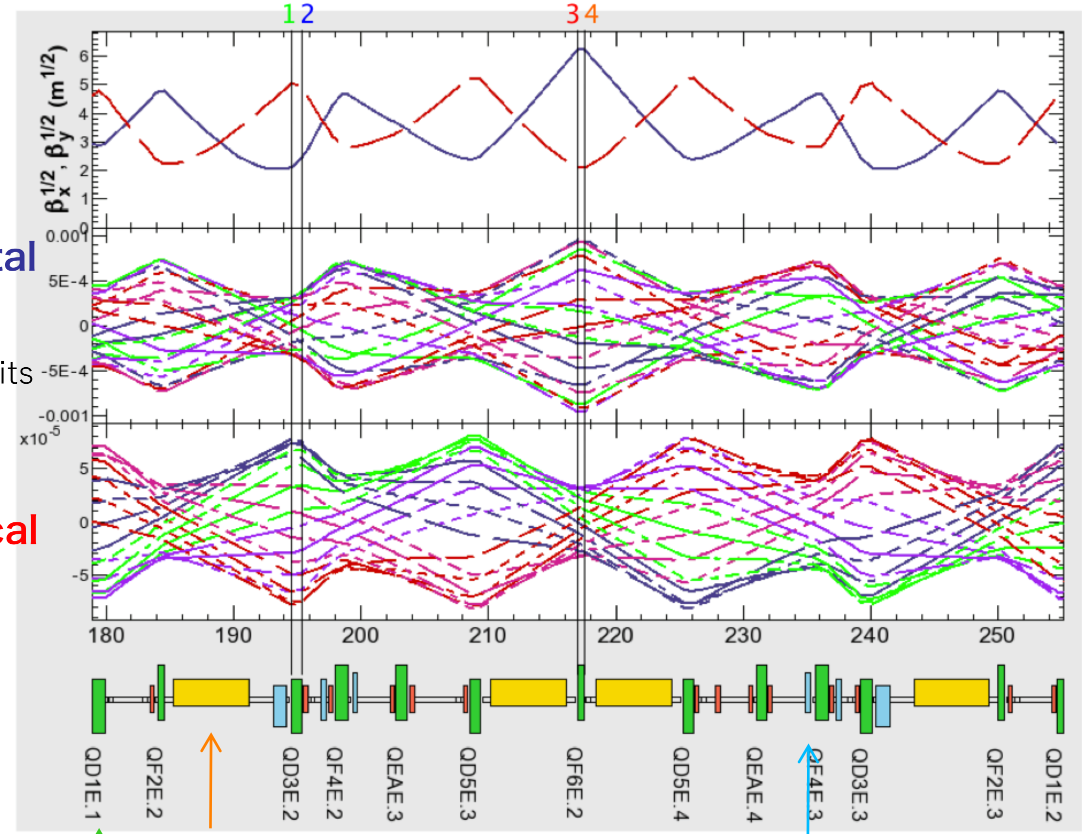
Betatron oscillation

horizontal

Beam particle orbits with same J and different phases

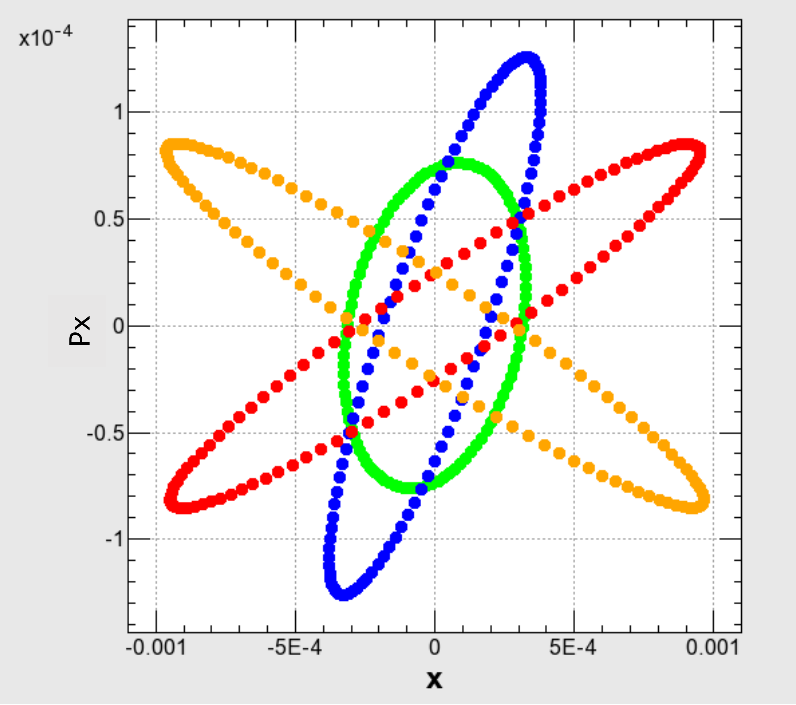
vertical



dipole

sextupole

quadrupole



Twiss parameter

- Given the Twiss parameters α_0 and β_0 at the entrance, the Twiss parameters and the phase difference ϕ at any location can be expressed using the transfer matrix as follows.

$$\alpha(s) = (M_{11}M_{22} + M_{12}M_{21})\alpha_0 - M_{11}M_{21}\beta_0 - M_{12}M_{22}\frac{1+\alpha_0^2}{\beta_0}$$

$$\beta(s) = -2M_{11}M_{12}\alpha_0 + M_{11}^2\beta_0 + M_{12}^2\frac{1+\alpha_0^2}{\beta_0}$$

$$\phi(s) = \arg(-M_{12}\alpha_0 + M_{11}\beta_0 + iM_{12})$$

$$\begin{aligned}
M &= \left[\begin{pmatrix} 1 & 0 \\ \alpha & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{\beta} & 0 \\ 0 & \sqrt{\beta} \end{pmatrix}\right]^{-1}\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ \alpha_0 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{\beta_0} & 0 \\ 0 & \sqrt{\beta_0} \end{pmatrix}\right] \\
&= \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\phi + \alpha_0\sin\phi) & \sqrt{\beta\beta_0}\sin\phi \\ -\frac{(\alpha-\alpha_0)\cos\phi + (1+\alpha\alpha_0)\sin\phi}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}}(\cos\phi - \alpha\sin\phi) \end{pmatrix} \\
&= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}
\end{aligned}$$

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Twiss parameter

- Consider a transfer matrix with short length *ds* and focusing power density *K*:

$$M = \begin{pmatrix} 1 & 0 \\ -Kds & 1 \end{pmatrix} \begin{pmatrix} 1 & ds \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & ds \\ -Kds & 1 - Kds^2 \end{pmatrix}$$

- Twiss parameters at $s = ds$:

$$\alpha(s) = (M_{11}M_{22} + M_{12}M_{21})\alpha_0 - M_{11}M_{21}\beta_0 - M_{12}M_{22}\frac{1+\alpha_0^2}{\beta_0} = \alpha_0 - \beta_0 K ds - \frac{1+\alpha_0^2}{\beta_0} ds + O(ds^2)$$

$$\beta(s) = -2M_{11}M_{12}\alpha_0 + M_{11}^2\beta_0 + M_{12}^2\frac{1+\alpha_0^2}{\beta_0} = \beta_0 - 2\alpha_0 ds + O(ds^2)$$

$$\phi(s) = \arg(-M_{12}\alpha_0 + M_{11}\beta_0 + iM_{12}) = \arg(-\alpha_0 ds + \beta_0 + i ds) = \frac{ds}{\beta_0} + O(ds^2)$$

- Take first-order terms:

$$\frac{d\alpha}{ds} = -K\beta - \frac{1+\alpha^2}{\beta}$$

$$\frac{d\beta}{ds} = -2\alpha$$

$$\frac{d\phi}{ds} = \frac{1}{\beta}$$

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IR optics

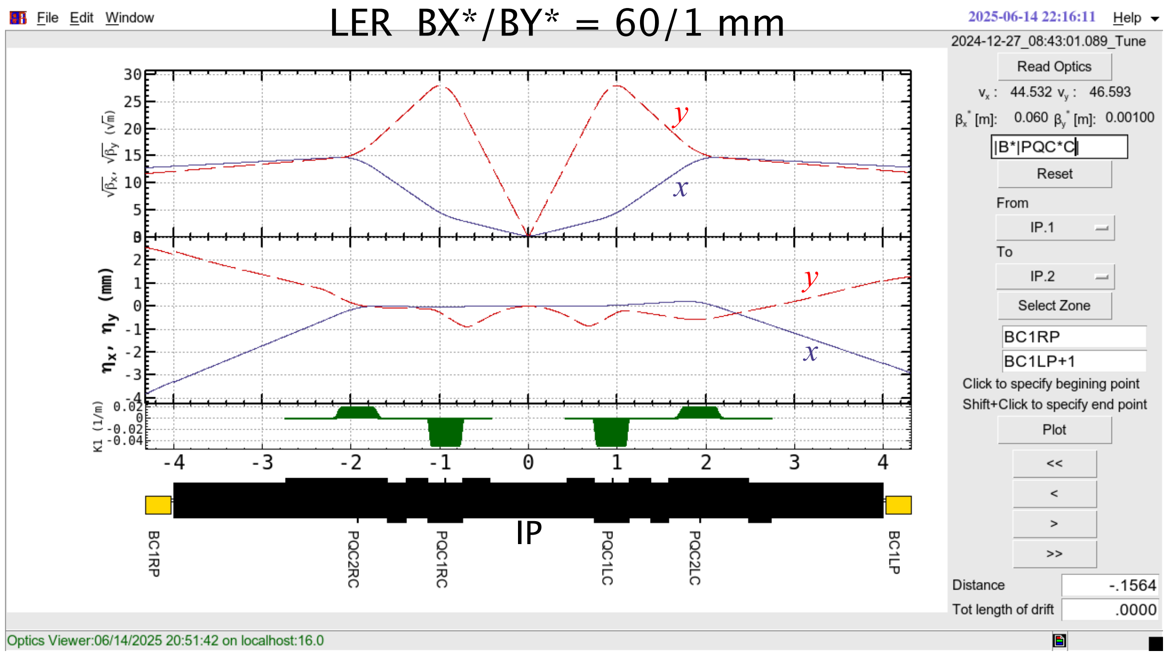
- In the drift space near the interaction point (IP), the beta function changes as follows:

$$M = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}, \qquad \alpha_0 = 0, \qquad \beta_0 = \beta^*$$

$$\begin{aligned} \alpha(s) &= (M_{11}M_{22} + M_{12}M_{21})\alpha_0 - M_{11}M_{21}\beta_0 - M_{12}M_{22}\frac{1+\alpha_0^2}{\beta_0} \\ &= -s/\beta^* \end{aligned}$$

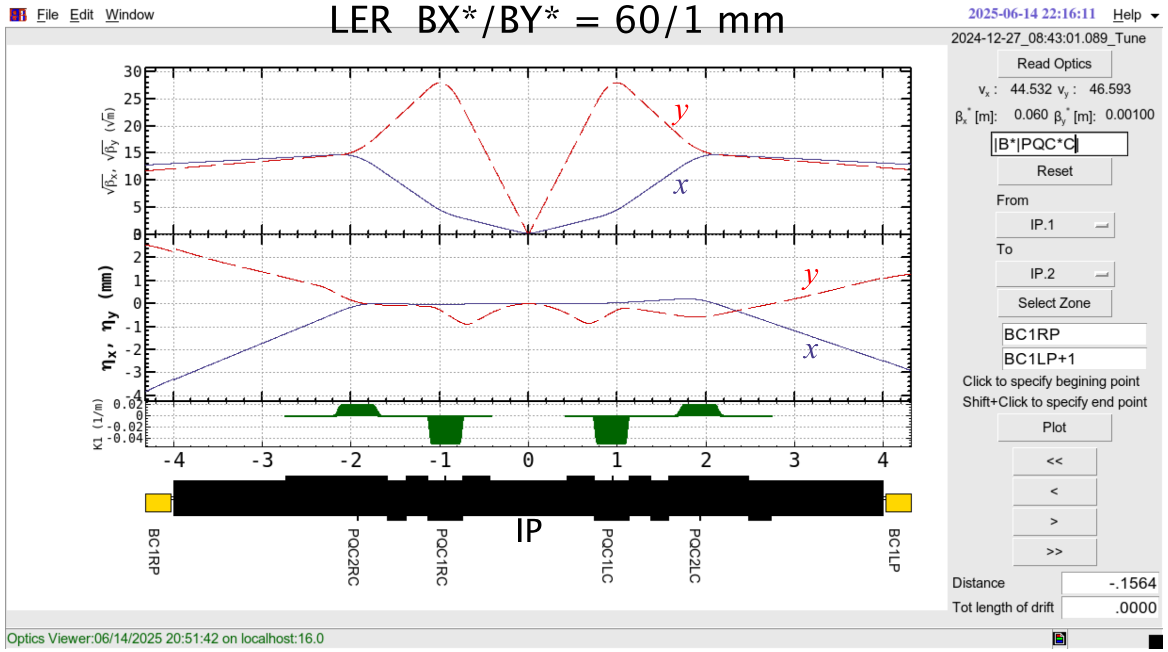
$$\begin{aligned} \beta(s) &= -2M_{11}M_{12}\alpha_0 + M_{11}^2\beta_0 + M_{12}^2\frac{1+\alpha_0^2}{\beta_0} \\ &\boxed{= \beta^* + s^2/\beta^*} \end{aligned}$$

$$\begin{aligned} \phi(s) &= \arg(-M_{12}\alpha_0 + M_{11}\beta_0 + iM_{12}) \\ &= \arg(\beta^* + is) \end{aligned}$$



IR optics

- The strength of the final-focus quadrupole magnet depends on the distance from the IP.
- The beta functions at the IP are adjusted without changing the strength of the final-focus quadrupole magnets.



IR optics

- On the arc side of QC1, β_y should be decreasing and therefore α_y should be positive. Simplified estimation using a thin quadrupole magnet is:

$$M = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix} \begin{pmatrix} 1 & l^* \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & l^* \\ -k & 1-kl^* \end{pmatrix}, \qquad \alpha_0 = 0, \qquad \beta_0 = \beta^*$$

$$\alpha(s_a) = (M_{11}M_{22} + M_{12}M_{21})\alpha_0 - M_{11}M_{21}\beta_0 - M_{12}M_{22}\frac{1+\alpha_0^2}{\beta_0}$$

$$= k\beta^* - \frac{l^*(1-kl^*)}{\beta*}$$

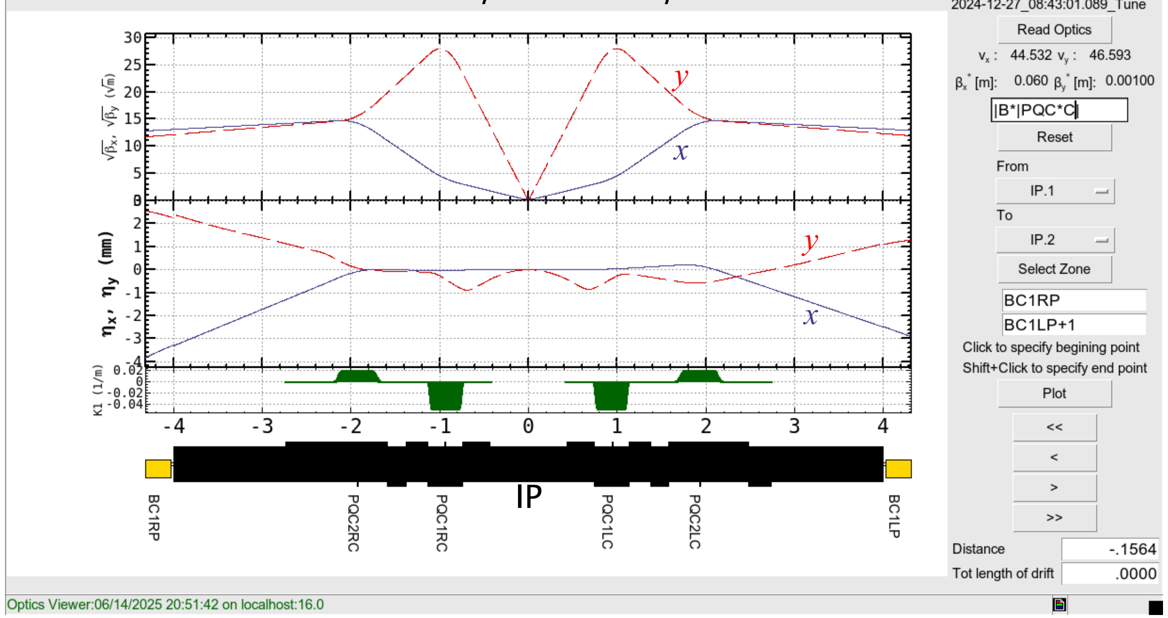
$$= \frac{1}{\beta*}(k(\beta^{*2} + l^{*2}) - l^*) > 0$$

- This means QC1 focusing strength should be:

$$k > \frac{l^*}{\beta^{*2} + l^{*2}} \sim \frac{1}{l^*}$$

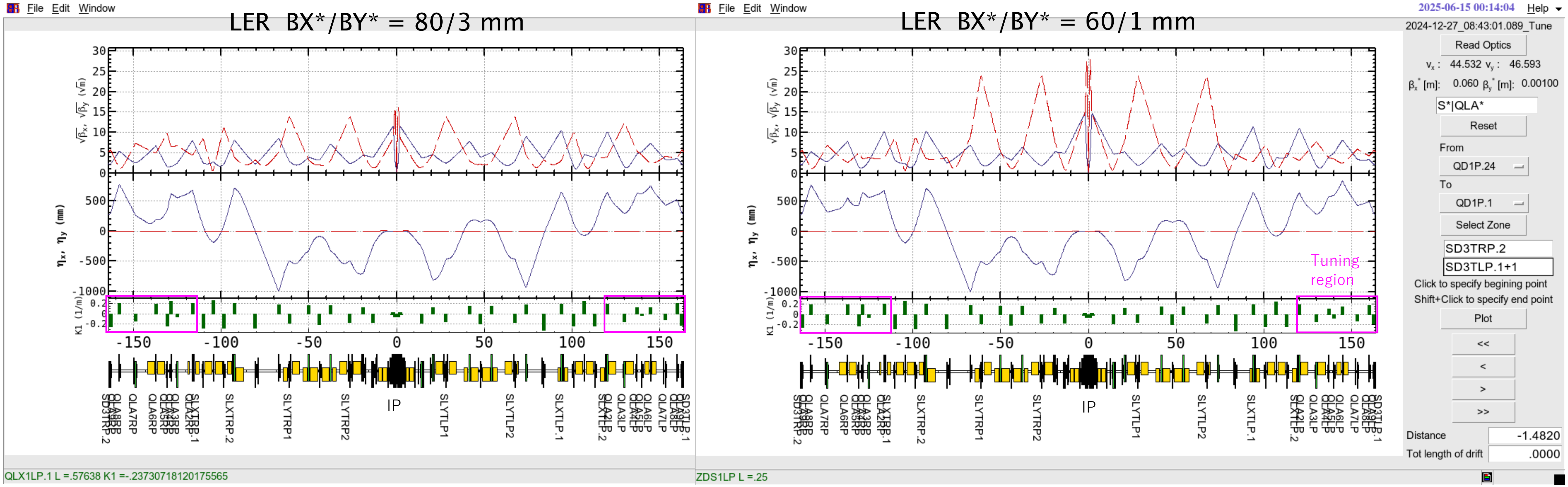
QC1LP |K1| = 1.722 1/m
L* from the QC1LP center = 0.935 m

LER BX*/BY* = 60/1 mm



IR optics

- To change the beta function of the IP, we adjust the quadrupole magnets in the tuning region on the arc side of the SLX pair, not the magnets on the collision point side. By using 8 magnets on each side, AX*, AY*, BX*, BY*, EX*, EPX*, δNX, δNY can be adjusted. Since the transfer matrix on the IP side is unchanged, the necessary conditions between IP, SLY, and SLX are preserved.



Dispersion, x-y coupling

- The transformation from the physical coordinate to the normal coordinate is given as:

$$\begin{pmatrix} X \\ P_X \\ Y \\ P_Y \end{pmatrix} = \begin{pmatrix} \mu & 0 & -r_4 & r_2 \\ 0 & \mu & r_3 & -r_1 \\ r_1 & r_2 & \mu & 0 \\ r_3 & r_4 & 0 & \mu \end{pmatrix} \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix} - \begin{pmatrix} \eta_X \\ \eta_{PX} \\ \eta_Y \\ \eta_{PY} \end{pmatrix} \Delta p$$

$$\mu^2 + (r_1 r_4 - r_2 r_3) = 1$$

- The dispersion and the x-y coupling will be discussed if there is another opportunity.

Thank you for your attention!