THE FIRST TRIAL OF XY-COUPLED BEAM PHASE SPACE MATCHING FOR THREE-DIMENSIONAL SPIRAL INJECTION*

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Abstract

The most recent measurement of muon $g-2$ results in a 3.8σ discrepancy with the equally precise theoretical prediction. The J-PARC muon $g-2$/EDM experiment (E34) is under preparation to decipher this discrepancy and unravel the new physics beyond the standard model. The precision goal for g - 2 is 0.1 ppm. To achieve this precision goal a novel three-dimensional spiral injection scheme has been devised to inject and store the beam into a small diameter MRI–type storage magnet for E34. The new injection scheme features smooth injection with high storage efficiency for the compact magnet. However, the spiral injection scheme is an unproven idea, therefore, a Spiral Injection Test Experiment (SITE) at KEK Tsukuba Campus is underway to establish this injection scheme. Due to the axial symmetric field of the solenoid magnet, a strongly XY-coupled beam is required. To produce the required phase space for the solenoid-type storage magnet, a beam transport line consisting of three rotatable quadrupole magnets has been designed and built for SITE. The vertical beam size reduction by means of phase space matching and other geometrical information has been successfully measured by the wire scanners. This report describe the first trial of phase space matching for the SITE.

INTRODUCTION

The most latest measurement of muon's $g-2$ at BNL [1] results in $3.8\,\sigma$ [2] discrepancy with the Standard Model prediction. This discrepancy could be a hint of new physics beyond the standard model. The new J-PARC muon $g-2$/EDM (E34) is under construction to measure muon $g-2$/EDM to unprecedented level. The E34 experiment will use a fully new approach in order to measure the muon's $g-2$/EDM. The final goal of E34 is to measure the muon's $g-2$ with a precision of 0.1 ppm and EDM down to the value of 10^{-21} e.cm [3, 4].

The muon's $g-2$/EDM will be measured by the means of low emittance muon beam storage into a precise magnetic field. In E34 a low emittance muon beam of momentum 300 MeV/c will be injected vertically into a compact 3–T Magnetic Resonance Imaging (MRI) type solenoid magnet, the muon beam will store with a 0.66 m diameter orbit. The use of MRI-type storage magnet gives an unprecedented local field uniformity of 0.2 ppm. A novel three–dimensional spiral injection scheme has been invented in order to inject the beam into the compact MRI–type magnet. This new injection scheme provides high injection efficiency and overcome technical challenges related to the compact nature of solenoid-type storage magnet.

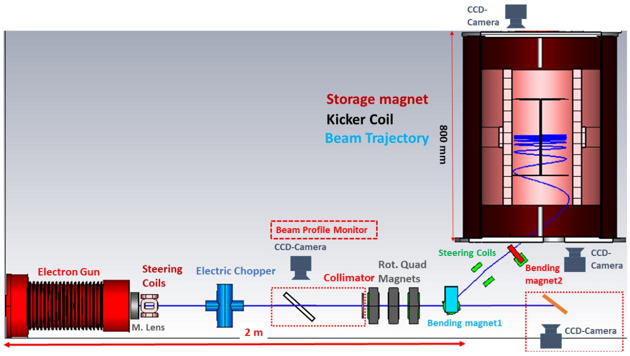

Figure 1: The 3-D model of Spiral Injection test Experiment (SITE) setup.

In the 3-D spiral injection scheme, the beam will be injected at the vertical angle into the storage magnet. The radial field of the solenoid will decrease the vertical angle of the beam as it approaches the mid plane of the magnet. Finally, a magnetic kicker will guide the beam to the storage volume where the beam will be stored under a weak focusing field. Due to the axial symmetric field of the solenoid magnet a so called "XY-coupled" beam is needed to avoid the vertical defocusing of beam inside the storage magnet [5].

The three-dimensional spiral injection scheme is an unprecedented injection idea, therefore, a demonstration experiment to establish the feasibility of this new injection scheme is inevitable. A scale down Spiral Injection Test Experiment (SITE) with an electron beam is under development at KEK Tsukuba campus. This paper will describe the first trial of beam injection with the XY-coupled beam into the solenoid storage magnet.

SPIRAL INJECTION TEST EXPERIMENT (SITE)

The SITE composed of a 2 m long straight beamline, a solenoid storage magnet for the electron beam storage and a bend section to inject the electron beam into the storage magnet. A triode type thermionic electron gun with LaB_6 cathode is used to generate the DC electron beam of 80 keV with the beam current in the range of a few μA. After the electron gun, a magnetic lens focuses the beam. A pair of steering coils also have been installed to control the trans-

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verse position of the beam. An electric chopper system after the electron gun produces a pulsed beam. Details of the electric chopper can be found in [6]. A collimator of diameter 3 mm and depth 5 mm is placed after the electric chopper. The collimator was used as the beam dump for the chopper system and also used to create the differential pumping system for the gas monitor in the storage magnet. After the collimator, three rotation quadrupole magnets have been installed for the beam phase space matching for the spiral injection. Due to the radial fringe field of the storage magnet, an XY-coupled beam at the injection point is required to avoid beam de-focusing.

After the rotation quadrupole magnets, a bending magnet is placed on the straight beamline to inject the beam at 40 degree towards the storage magnet. A second bending magnet was installed near the injection point for the slight control of the injection angle.

The storage magnet is basically a solenoid which consists of a 540 mm long solenoid coil. The solenoid coil is enclosed in the iron yoke. An auxiliary coil was placed at the center of the solenoid in order to produce a weak focusing field at the center of the storage magnet [7]. The pulsed magnetic kicker will keep the beam to the center of the storage magnet [8]. The layout of the SITE experimental setup is shown in Fig. 1. A new wire scanner monitor for the beam profile and position measurement was developed for the solenoid storage magnet [9]. The vertical beam blow has been successfully suppressed by the phase space matching. The details will be described in the forthcoming sections. A comparison of parameters between E34 and SITE is given in Table 1.

Table 1: Comparison of Parameters between E34 and SITE.

Parameters	E34	SITE
Magnetic field strength	3 T	0.0082 T
Momentum	300 MeV/c	296 keV/c
Cyclotron Period	7.4 ns	5 ns
Storage orbit diameter	0.66 m	0.24 m

THE MATCHING BEAMLINE

One of the key requirement of the 3-D spiral injection scheme is to produce the required beam phase-space for the injection to avoid the vertical beam blow–up. Here, we used a transfer matrix method to compute the coupling of the beam at the matching point. Let's assume a matrix M which consists of rotation and Twiss parameters matrix [5, 10]:

$$M = U_{out}^{-1} \cdot D \cdot U_{in}, \tag{1}$$

where, U_{in} and U_{out} are input (initial) and output (matching point) rotation matrix, which satisfy the symplectic condition. In the case of, uncoupled input beam phase-space U_{in} will be equal to identity matrix. In the case of SITE, the input beam phase-space has no coupling, therefore, Eq. (1) can be written as follow:

$$M = U^{-1} \cdot D, \tag{2}$$

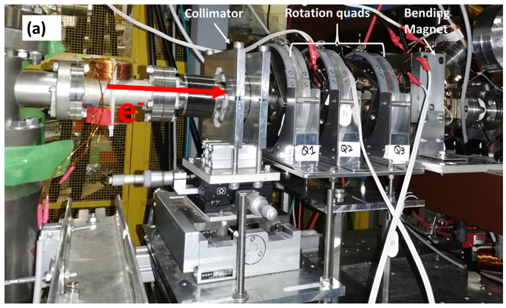

Figure 2: The layout of the matching beamline, the beamline composed of three rotatable quadrupole magnets, a rectangular bending magnet and drift space. The rotatable quadrupole magnets were placed in the downstream direction of the collimator.

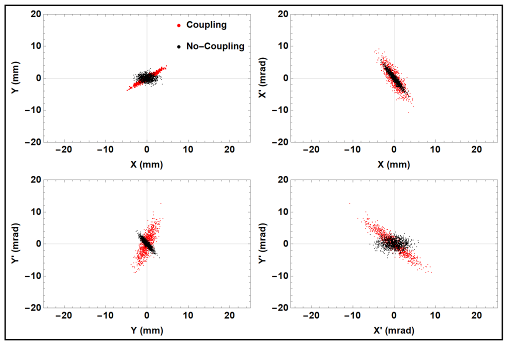

Figure 3: The beam phase space with and without XY-coupling.

The 4×4 U and D matrices are given as follow:

$$U = \begin{pmatrix} \mu & 0 & -R_4 & R_2 \\ 0 & \mu & R_3 & -R_1 \\ R_1 & R_2 & \mu & 0 \\ R_3 & R_4 & 0 & \mu \end{pmatrix}, \quad D = \begin{pmatrix} D_X & 0 \\ 0 & D_Y \end{pmatrix}, \tag{3}$$

where, R_1, R_2, R_3, R_4 are known as coupling parameters and non-zero value of these parameters shows the strength of the coupling. The quantity μ is given as $\mu = \sqrt{1 - (R_1 R_4 - R_2 R_3)}$. The matrix D is a diagonal matrix consist of beam input and output Twiss parameters.

An iterative procedure has been adopted to calculate the coupling parameters. The emittance of the beam for SITE at the collimator location was measured by the quadrupole scan method. The values of horizontal (ϵ_x) and vertical emittance (ϵ_y) are 0.61 ± 0.05 mm–mrad and 0.41 ± 0.04 mm–mrad, respectively [11].

The matching beamline consists of three rotatable quadrupole magnets, a bending magnet, and drift spaces. Figure 2 presents the photo matching beamline. These three rotatable quadrupole magnets can be rotated at an arbitrary angle around the beam. The transfer matrix of the beam transport line can be written as follow:

$$M = L_5 \cdot B \cdot L_4 \cdot Q_3 \cdot L_3 \cdot Q_2 \cdot L_2 \cdot Q_1 \cdot L_1 \tag{4}$$

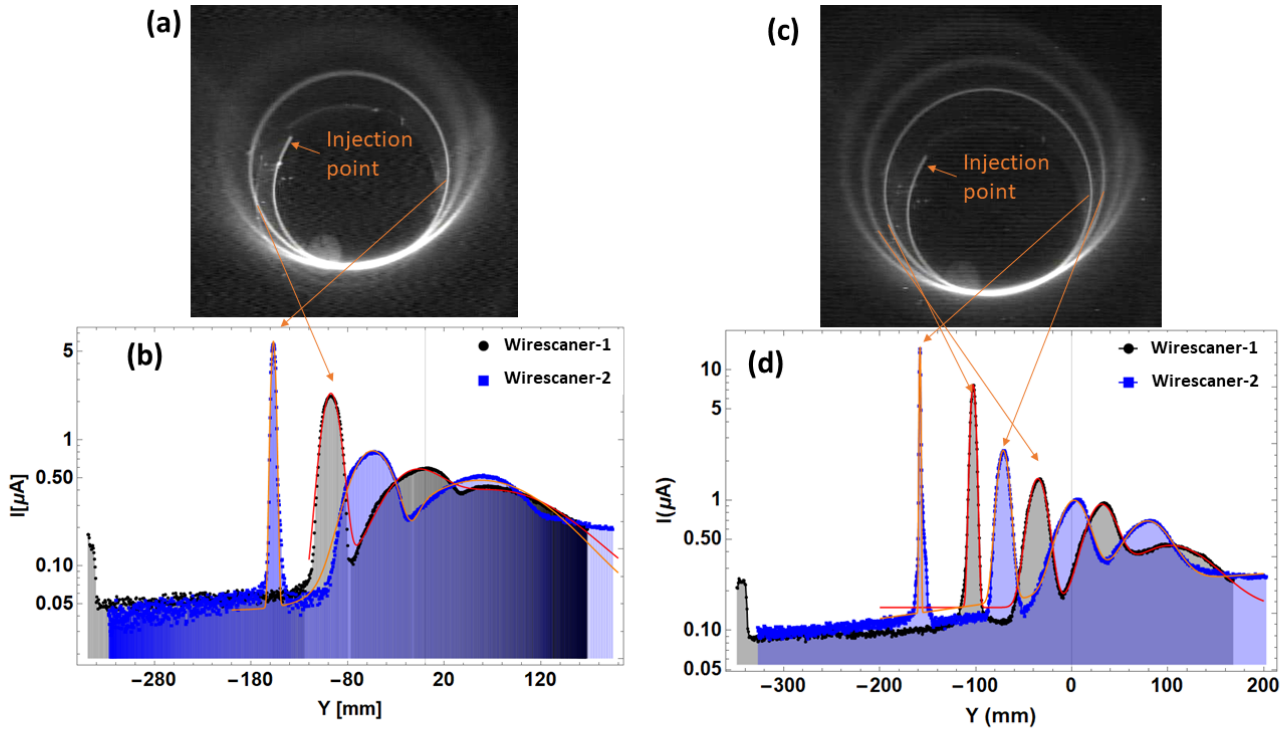

Figure 4: (a) The gas monitor view of the beam injection into the storage magnet without XY-coupling. (b) The vertical beam size and position are measured by the wire scanner in the case of no coupling. The horizontal axis is the vertical position of the wire scanner in the storage magnet and vertical axis is beam current deposited at the wire scanner. It is very evident that beam blow-up drastically without appropriate phase space matching. (c) The gas monitor results with XY-coupling. (d) The vertical beam size was measured by the wire scanner with XY-coupling.

where, M is a transport line matrix, B is bending magnet, D represents the drift space, Q_1, Q_2, and Q_3 denotes the rotation quadrupole magnets. A custom program in Mathematica was developed to find the appropriate rotation angle and strength of the rotation quadrupole magnets. The black dots in Fig. 3 shows the beam phase space without XY–coupling and red dots are corresponds to the coupled the beam phase. In the case of coupled beam phase space the rotation angles of Q_1, Q_2, and Q_3 were -20 deg, 25 deg and -45 deg respectively.

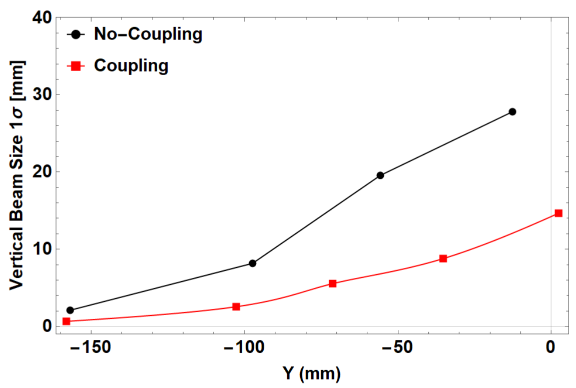

Figure 5: The comparison of vertical beam size with and without XY-coupling. The vertical beam size at the kick point (Y ≈-100 mm) was 8.18±0.03 mm (1σ) without XY–coupling, which has been controlled to 2.56±0.005 mm (1σ) with appropriate phase space matching.

Figure 4 (a) and (b) shows gas monitor and wire scanner results of the beam injection without any phase space matching, respectively, the beam blow–up in the vertical direction is very clear from gas monitor and wire scanner measurement. Whereas, Fig. 4 (c) and (d) gas monitor and wire scanner results of the beam injection with the appropriate XY-coupling as shown in Fig. 3 (red dots), respectively. The reduction in the vertical beam size is visible by the gas monitor and quantitative information can be seen by the wire scanner. Without XY-coupling only 2–turns can be seen by the gas monitor, whereas, when the beam was injected with appropriate XY-coupling 4–turns can be seen very clearly. Figure 5 shows the comparison of vertical beam size with and without XY-coupling.

CONCLUSION

In order to overcome the vertical beam blow–up in the solenoid-type storage magnet, a beam transport line consisting of three rotatable quadrupole magnets has been designed and built for SITE. The dramatic reduction in the vertical beam size was observed with the appropriate setting of three rotatable quadrupole magnets. Without any beam phase–space matching, the beam size grows to 8.18±0.03 mm (1σ) at the kick point (Y ≈-100 mm) of the storage magnet. The vertical beam blow–up was reduced to 2.56±0.005 mm (1σ) at the kick point with the appropriate combination of the three rotation quadrupole magnets.

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REFERENCES

[1] G.W. Bennett *et al*., "Final Report of the muon E821 anomalous magnetic moment measurement at BNL", Phy. Rev. D., vol. 73, p. 072003, 2006. `doi:10.1103/PhysRevD.73.072003`.

[2] Alexander Keshavarzi, Daisuke Nomura, and Thomas Teubner , "$g-2$ of charged leptons, $\alpha(M_Z^2)$, and the hyperfine splitting of muonium", Phys. Rev. D., vol. 101, p. 014029, 2020. `https://link.aps.org/doi/10.1103/PhysRevD.101.014029`.

[3] M. Abe *et al*., "A new approach for measuring the muon anomalous magnetic moment and electric dipole moment", Progress of Theoretical and Experimental Physics, vol. 2019, `https://doi.org/10.1093/ptep/ptz030`.

[4] M. Aoki *et al*., "Conceptual Design Report for the Measurement of the Muon Anomalous Moment $g-2$ and Electric Dipole moment at J-PARC (2011)", unpublished.

[5] H. Iinuma *et al*., "Three-dimensional spiral injection scheme for the $g-2$/EDM experiment at J-PARC", Nucl. Instr. Meth., vol. A832, pp. 51-62, 2016. `doi:10.1016/j.nima.2016.05.126`.

[6] M. A. Rehman *et al.* "Development of the three-dimensional injection spiral injection by using electron beam for muon $g-2$ experiment", in *Proc. of 14th Annual Meeting of Particle Accelerator Society of Japan (PSAJ'17)*, Sapporo, Japan, Aug. 2017, paper WEOM01, pp. 114–117.

[7] M. A. Rehman *et al.*, "Progress of the Novel Three-dimensional Spiral Injection Scheme Test Experiment", in *Proc. 29th Linear Accelerator Conf. (LINAC'18)*, Beijing, China, Sep. 2018, pp. 717–720. `doi:10.18429/JACoW-LINAC2018-THPO017`.

[8] M. A. Rehman *et al.*, "Development of the pulsed magnetic kicker for the spiral injection test experiment", *Proc. of 15th Annual Meeting of Particle Accelerator Society of Japan (PASJ2018)*, Nagaoka, Japan Aug. 2018, paper THP006, pp.768-772.

[9] M. A. Rehman *et al*., "Beam Control and Monitors for the Spiral Injection Test Experiment", Journal of Physics: Conference Series, vol. 1350, p. 012151, 2019. `10.1088/1742-6596/1350/1/012151`.

[10] K. Oide, "Introduction to beam dynamics", in Japanese. 1992, `https://www-jlc.kek.jp/2003oct/subg/ir/lib/DOC/japanese/Lecture.beam.dynamics.pdf`.

[11] M. A. Rehman, Doctoral Thesis, `http://id.nii.ac.jp/1013/00006023/`.