

# A MAGNETIC CIRCUIT DESIGN AND PERFORMANCE EVALUATION OF NOVEL UNDULATOR AS A LIGHT SOURCE

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## Abstract

A novel undulator, called Leaf undulator, can provide linearly polarized radiation. The Leaf undulator can suppress the on-axis power density by an order of magnitude compared to Knot and Figure-8 undulators. Because the transverse velocity of electron beam in this undulator switches its direction from right to left in every undulator period. This paper shows a magnetic circuit design and performance evaluation of Leaf undulator as a light source.

## 1. Introduction

Undulator is able to provide a quasi-monochromatic synchrotron radiation with periodic magnets, significantly higher brightness than that from a bending magnet. In a basic linear undulator, electrons move on sinusoidal orbit with sinusoidal magnetic field. Therefore, it can produce linearly polarized radiation.

Electron orbit is:

$$x = -\frac{eB_0\lambda_u^2}{4\pi^2\gamma m} \sin(2\pi y/\lambda_u) = -\frac{2\pi K}{\gamma\lambda_u} \sin(2\pi y/\lambda_u),$$

where,  $K=0.934B_0[\text{T}]\lambda_u[\text{cm}]$  which is called the undulator parameter,  $B_0$  is peak magnetic field and  $\lambda_u$  is the period length. For an undulator with a large number of periods, interfere effects in the radiation produces essentially many collinear source points, and hence the radiation wave length is given by:

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right),$$

where,  $\theta$  is the angle of observation from beam axis, and  $n$  is the harmonic number. The radiation power on-axis is given by:

$$\frac{d^2P(\omega)}{d\omega d\Omega} \Big|_{\theta=0} = \frac{e^2\gamma^2}{4\pi\epsilon_0 c} \sum_n F_n^{(P)}(K) H_n \left( \frac{\omega}{\omega_l} \right)$$

$$H_n \left( \frac{\omega}{\omega_l} \right) = \frac{\sin^2(N\pi((\omega/\omega_l)-n))}{(\pi((\omega/\omega_l)-n))^2}$$

$$F_n^{(P)}(K) = \frac{n^2}{(1+K^2/2)^2} |E(\theta=0)|, (n: \text{odd})$$

$$F_n^{(P)}(K) = 0, (n: \text{even})$$

where,  $H_n$  is the spectrum shape function, and  $F_n$  is the spectrum intensity function. For  $K<1$ , only the fundamental is important. For  $K=1$  the power in the fundamental is maximum and few harmonics have appreciable intensity on-axis. For  $K>1$  more harmonics appear. In a helical undulator, electrons move on spiral orbit in combination with a vertical sinusoidal magnetic field and a horizontal one, having a phase difference of 1/4 period. The radiation power on-axis is given by:

$$\frac{d^2P(\omega)}{d\omega d\Omega} \Big|_{\theta=0} = \frac{e^2\gamma^2}{4\pi\epsilon_0 c} \sum_n F_n^{(h)}(K_x, K_y) H_n \left( \frac{\omega}{\omega_l} \right) \quad \omega_n = \frac{8\pi cn\gamma^2}{\lambda_0(2+K_x^2+K_y^2)}$$

$$F_n^{(P)}(K_x, K_y) = \frac{n^2}{(1+(K_x^2+K_y^2)/2)^2} |E_x^2(\theta=0) + E_y^2(\theta=0)|.$$

In this case only the fundamental appears. On a high-energy storage ring like SPring-8, for  $K>1$  quite a few harmonic radiation powers on-axis are obstructive to provide wanted fundamental linearly polarized radiation. Therefore we need a Leaf undulator to provide linearly polarized radiation by adding two opposite circular polarizations in each alternating period with low power density on-axis. A single period of ideal magnetic field model of leaf undulator is shown in Fig.1[1]. Fig.1 (a) shows that the electrons in a Leaf undulator pass, an inclined Figure-8 trajectory in the transverse plane. Then, due to the interference between the left and right-circularly polarized radiation, the resulting radiation carries linear polarization. Vertical magnetic field and horizontal magnetic field have  $\pi/2$  phase shift alternatively. Therefore, corresponding with left-circularly polarized radiation and the right-circularly polarized one, the final radiation carries linear polarization. The velocity projection of the electrons in the transverse plane is like a leaf and thus it is called Leaf undulator. These simulations are performed using SPECTRA 8.0[2] based on the storage ring parameters of SSRF[3]. A performance comparison of Leaf undulator, Knot undulator[4], Figure-8 undulator[5], and conventional linear undulator is shown in Table.1, where we assume the same undulator length (6.6 m, the length corresponding to minimum common multiple about the periods of different undulators), adjusted their peak fields to same fundamental energy of 7 eV.

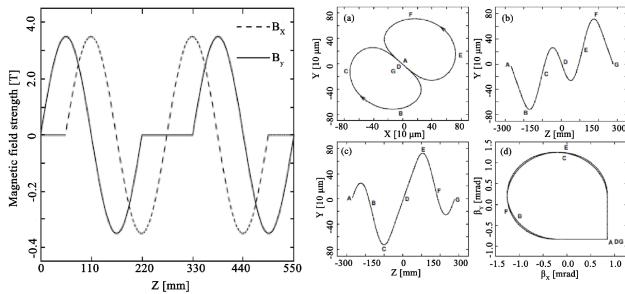


Figure 1: Magnetic field model of leaf undulator, a period, period length is 550 mm, peak field is 0.35 T (a)Trajectory in x-y plane (b)In y-z plane (c)In x-z plane (d)Velocity in x-z plane, showing leaf like carves (courtesy by professor Qiao)

Table 1: A performance comparison of leaf undulator, Knot undulator, Figure-8 undulator, and conventional linear undulator.

Undulator type	Leaf	Knot	Figure-8	Linear
Beam energy [GeV]	3.5	3.5	3.5	3.5
Length [m]	6.6	6.6	6.6	6.6
Fundamental energy [eV]	7	7	7	7
Period length X/Y [cm]	55/55	22/23	44/22	0/22
Peak field X/Y [T]	0.35/0.35	0.29/0.5	0.21/0.42	0/0.59
Total power [W] (in 0.6×0.6 mrad <sup>2</sup> )	0.268	3.247	5.04	363.167
Photon flux of fundamental [ $10^{14}$ ph/s 0.1% B.W.] (in 0.6×0.6 mrad <sup>2</sup> )	2.865	3.199	2.278	3.249

## 2. A magnetic circuit design

In this paper, we designed cascaded array of magnet for a leaf undulator by using RADIA[6]. This magnet arrangement has an empty space of 3/10 period in each period to introduce a phase shift. Thus, vertical magnetic field and horizontal magnetic field have  $\pi/2$  phase shift of sine wave alternatively. If the magnet array is capable to slide its position to have spacing of 1/8 (one-magnet-block space) period, it can generate helical magnetic field, so it is able to provide circular polarizations. Three periods of magnet arrangement for a Leaf undulator and helical mode are shown in Fig.2. The scale of a permanent magnet is 15×25×30 mm<sup>3</sup>, and a period length is 250 mm, magnetic material is NdFeB, a gap is 20 mm. An arrow in each magnet indicates magnetic field directions. In the calculation, magnet arrangement has 10 periods with compensation magnet at the either end. The magnetic fields in vertical and horizontal direction are shown in Fig.3. Peak field is about 0.3 T. Fig.3 shows trajectory in xz plane and velocity in x-z plane showing leaf like curves, calculated by B2E[7].

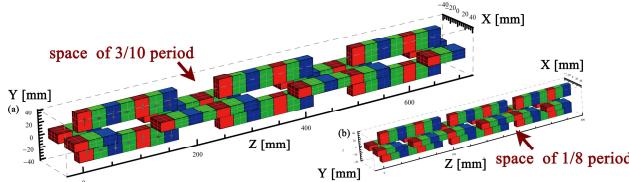


Figure 2: (a)Three periods of magnet circuit design for leaf undulator (b)for helical mode

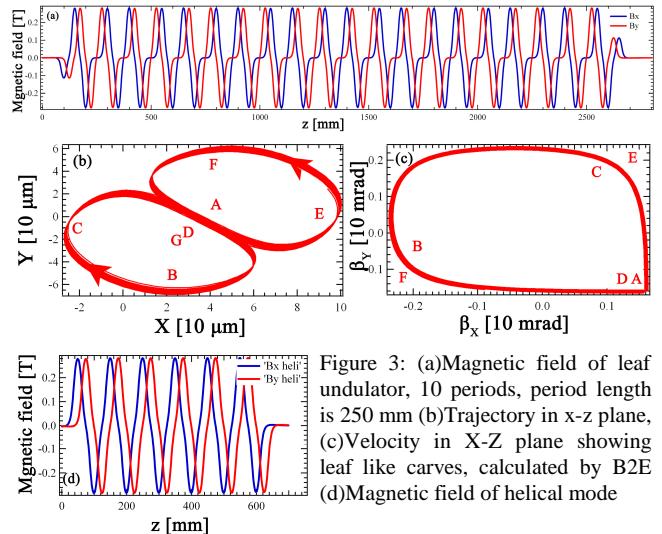


Figure 3: (a)Magnetic field of leaf undulator, 10 periods, period length is 250 mm (b)Trajectory in x-z plane, (c)Velocity in X-Z plane showing leaf like carves, calculated by B2E (d)Magnetic field of helical mode

## 3. Calculation of radiation spectrum

We examined the performance of this Leaf undulator by assuming it is used on HiSOR[8] storage ring with 0.7 GeV beam energy using SPECTRA 8.1. In Fig.4, it is shown that 5 eV radiation is fundamental one, and some higher order harmonics appeared. As shown in Fig.1, the period of electrons' transverse oscillation along 45° directions (corresponding to 45° polarization) is twice larger than as that in 135° direction (corresponding to 135° polarization). Therefore, the transverse oscillation along 45°direction generates the 45° linearly polarized harmonic with a half photon energy of fundamental and its higher harmonics on the half-odd-integer appear in the spectrum. The spatial power density distribution of the radiation generated from the Leaf undulator and its 5 eV fundamental's flux distribution are shown in Fig.4 which indicates that the majority of radiation power distributes off-axis of the undulator axis as expected, while the fundamental's flux is mainly concentrated on the undulator axis.

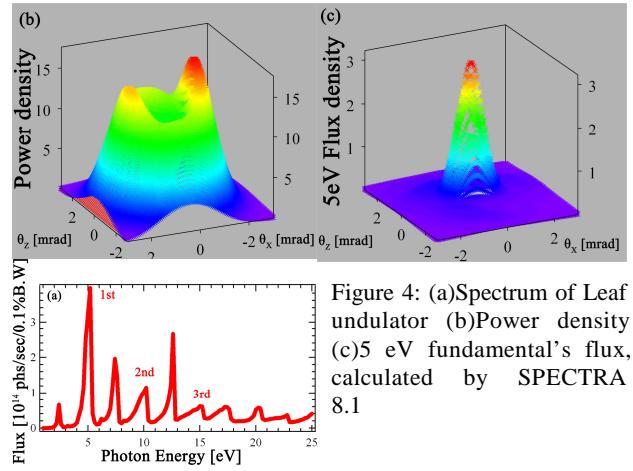


Figure 4: (a)Spectrum of Leaf undulator (b)Power density (c)5 eV fundamental's flux, calculated by SPECTRA 8.1

## **4. Conclusion**

A novel undulator, called Leaf undulator, can provide linearly polarized fundamental radiation without other dominant higher order radiations on-axis. If magnet can slide to have space of 1/8 period, it can generates helical magnetic field, so it is able to provide circular polarizations, as well.

## **Reference**

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