THE IMPROVEMENT OF THE AUTO-TUNING SYSTEM IN KEK-PS MAIN RING RF CAVITY

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## ABSTRACT

The performance of the auto-tuning system in KEK-PS main ring was analyzed, and the system has been improved. The error in resonance frequency is reduced to one tenth of that of the old system.

## INTRODUCTION

Ferrite loaded cavities are commonly used in the proton synchrotron. The resonant frequency of the cavity is varied by the bias current which changes the permeability of the ferrite. The structure of the cavity used in KEK-PS is illustrated in Fig. 1, where the flow of the bias current is shown by the arrows on the lines. The dotted lines with arrows show the flow of the RF current. The operating point on the B-H curve of the ferrite is determined by the bias current.

The cavity resonates at the frequency satisfying the relation $\ell_0/2$ = (wave length)/4, and the light velocity in the ferrite is given by $1/\sqrt{\varepsilon\mu}$. If we assume that the cavity is filled with the ferrite, the resonant frequency of the cavity is written as

$$\Omega_0 = \frac{\pi}{\ell_0\sqrt{\varepsilon\mu}} \quad ,$$

where $\varepsilon$ is the dielectric constant of the ferrite, and $\mu$ its permeability.

The effect of the bias current on the resonant frequency of the cavity is not linear. But in this analysis, we use a linear relation between them for small amplitude variation in bias current. In order to explain the relation, we use the experimental results.

When the bias current Ib is increased by $\Delta$Ib at t = 0, the variation in $\mu$ is approximated with the following equation (Fig. 2);

$$\Delta\mu(t) = -\,K\Delta Ib(1 - S\exp(-\,t/T)) \quad ,$$

where $K \simeq 1.6\ \mu_0$ 1/A, $S \simeq 0.15$, $T \simeq 0.09$ sec.

for $\Delta$Ib = 100 A.

When Ib is decreased by 100 A, S is negative, i.e., there is an excessive increment in $\mu$. In this case, the permeability $\mu$ is varied by Ib with no retardation.

Though the above equation is a result for $\Delta$Ib = + 100 A, we assume that the retardation, caused by the ferrite, is represented by the equation describing the case where the bias current is increased. Taking the differentiation of the above equation with respect to time,

$$\frac{d\Delta\mu(t)}{dt} = -\,(K\Delta Ib + \Delta\mu(t))/T \quad .$$

Consisder the case where $\Delta$Ib has the time dependence $\exp(j\omega t)$, then $\Delta\mu(t)$ must also have the same dependence on time. Therefore

$$\Delta\mu(\omega) = -\,K\Delta Ib(\omega)/(1+j\omega T) \quad .$$

The equation shows that the phase difference between $\Delta\mu(\omega)$ and $\Delta Ib(\omega)$ is 90° for the frequencies higher than 1/T(1 $\sim$ 2 Hz). We assume that $\mu$ is given by the sum of its mean value and the incremental part cused by $\Delta Ib(\omega)$,

$$\mu = \mu_I - K\Delta Ib(\omega)/(1+j\omega T) \quad ,$$

where $\mu_I$ is the value of $\mu$ at bias current Ib.

Hence the resonant frequency of the cavity is written as

$$\Omega_0 = \frac{\pi}{\ell_0\sqrt{\varepsilon\mu_I}}\left[1-(K/\mu_I)\Delta Ib(\omega)/(1+j\omega T)\right]^{-1/2}$$

$$= \Omega_I\left[1-(K/\mu_I)\Delta Ib(\omega)\right]^{-1/2}\cdot\left[(1+j\omega T)/(1+j\omega T')\right]^{1/2},$$

where $\Omega_I = \pi/(l_0\sqrt{\varepsilon\mu_I})$ is the resonant frequency at bias current Ib, and

$$T' = T/\left[1-(K/\mu_I)\Delta Ib(\omega)\right] \quad , \qquad (T' > T) \quad .$$

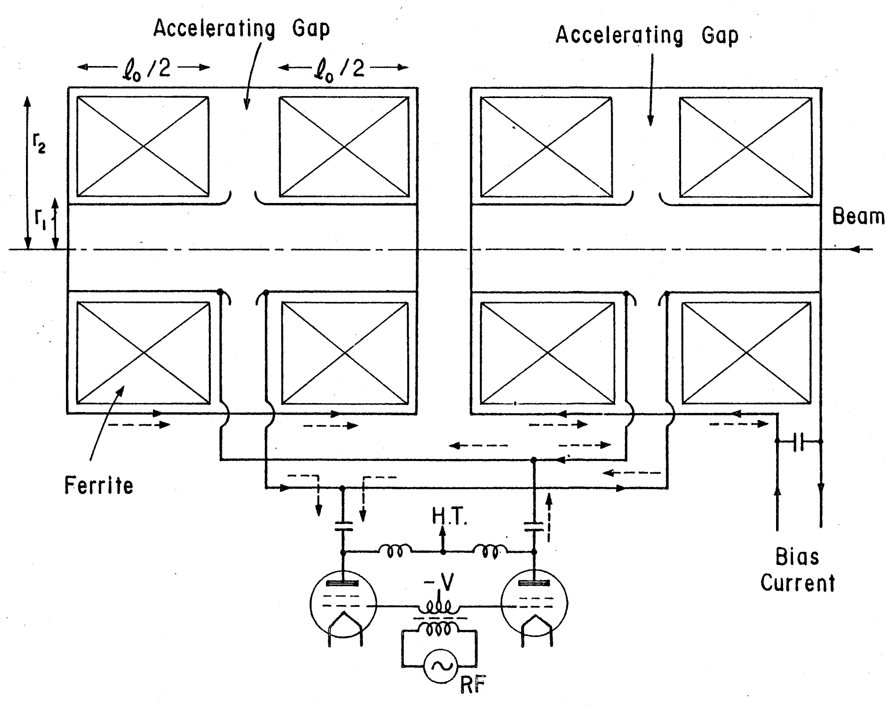


Fig. 1 The structure of the RF cavity at KEK-PS main ring.

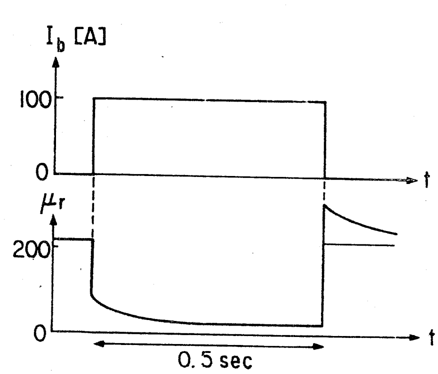


Fig. 2 The variation in $\mu_r$.

When $\omega << 1/T'$ (i.e., less than 2 Hz), the imaginary parts in the equation of $\Omega_0$ can be neglected, and

$$\Omega_0 = \Omega_I\left[1 + K\Delta Ib(\omega)/(2\mu_I)\right] \quad ,$$

or $$\Delta\Omega_0(\omega) = \Omega_I K\Delta Ib(\omega)/(2\mu_I) \quad ,$$

where $\Delta\Omega_0(\omega)$ is the incremental part of $\Omega_0$.

In the case of $\Delta Ib(\omega) << \mu_I/K$, $T' \simeq T$ and the equation of $\Delta\Omega_0(\omega)$ also valid for the frequencies higher than $1/T$.

Hence there is no phase difference between $\Delta\Omega_0(\omega)$ and $\Delta Ib(\omega)$ for small amplitude variation in Ib. Though we used the rough approximations in the discussion, the result is correct for the frequency range up to 1 kHz (see Fig. 4). For large amplitude, the relation between them is not so simple as the above discussion.

## THE ANALYSIS OF THE AUTO-TUNING SYSTEM

In this analysis, a simple model, which is diagrammatically shown in Fig. 3, is used. If the bias current is ideally programmed, the signals shown in the diagram are zero. The symbol $\Delta\Omega$ is the error in frequency between applied RF and resonance of the cavity.

The estimation of the reduction of $\Delta\Omega$ is the purpose of this analysis.

### Correction amplifier

This amplifier is the compensation network for the auto-tuning system.

The transfer function of this amplifier is written as;

$$F_c(j\omega) = \frac{G_c\omega_1}{j\omega + \omega_1}\,\frac{j\omega + \omega_2}{\omega_2},$$

where $\omega_1 = 2\pi \times 10$ rad./sec,

$\omega_2 = 2\pi \times 8 \times 10^2$ rad./sec,

and $G_c = 80$ V/V.

The value of $\omega_1$ determines the loop gain at lower frequency. Since loop gain at lower frequency is sufficiently large, we limit the increase in loop gain to the value at 10 Hz. And $\omega_2$ is introduced to compensate the phase lag at higher frequency.

The former system had complicated compensation networks at high frequencies, and individual adjustments of the parameters were not possible.

### Bias-current power supply

Readjustment of the bias-current power supply was necessary to make the system most suitable condition. The transfer function of the power supply can be approximated by

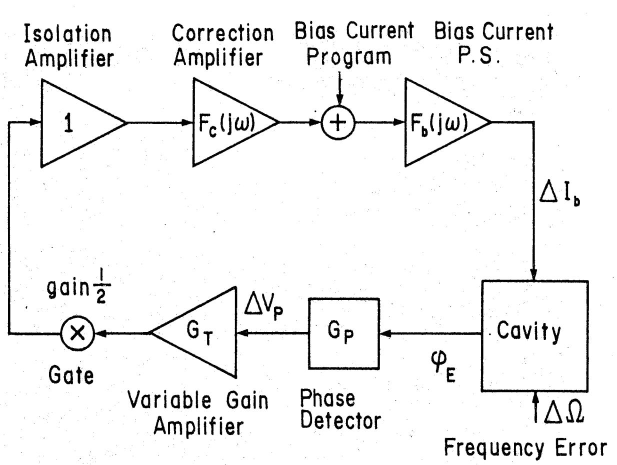


Fig. 3 The block diagram of the auto-tuning system.

$$F_b(j\omega) = \frac{G_b\omega_0^2}{(j\omega)^2 + 2\eta\omega_0 j\omega + \omega_0^2}$$

where $G_b = 100$ A/V, $\eta = 0.5$,

and $\omega_0 = 2\pi \times 2.1 \times 10^4$ rad./sec.

Before the adjustment, the parameters of this power supply were $\eta \simeq 0.3$ and $\omega_0 \simeq 2\pi \times 4k$ rad./sec. The frequency response had a maximum at 4 kHz and decreased at the frequencies higher than 4 kHz. Since the phase characteristics changed abruptly near the peak frequency, the phase lag of the former power supply was difficult to be compensated with the other circuits.

### The relation between $\Omega_0$ and Ib

The relation between the resonant frequency $\Omega_0$ of the cavity and bias current Ib is not clear. If we limit our analysis within a small amplitude oscillation in Ib, the relation between $\Omega_0$ and Ib can be approximated by the linear function with no retardation as mentioned in the previous section. Since the variation in Ib from 100 A to 400 A causes the change in resonant frequency from 6 MHz to 8 MHz,

$$\frac{\Delta\Omega_0}{\Delta Ib} = 4.2 \times 10^4 \text{ (rad./sec)/A} \quad .$$

### The relation between $\phi_E$ and $\Delta\Omega$

The symbol $\phi_E$ means the phase difference between the applied RF voltage of the cavity and its current. The characteristics of the cavity are equivalent to that of the R-L-C parallel network in the vicinity of the resonant frequency. When the cavity is tuned to resonate at the applied RF frequency, $\phi_E = 0$. If there is an error in resonant frequency given by $\Delta\Omega = \pm\Omega_0/(2Q)$, $\phi_E$ takes the value of $\mp\pi/4$ rad., where $Q(=R/(\Omega_0 L))$ is the quality factor of the cavity. Hence in linear approximation,

$$\phi_E = -\frac{\pi}{2}\,Q\,\frac{\Delta\Omega}{\Omega_0} \quad ,$$

where $\Delta\Omega = \Omega - \Omega_0$.

### Phase detector

The phase detector detects $\phi_E$, and gives the error voltage $\Delta V_p$ with sensitivity $G_p$;

$$\Delta V_p = G_p\phi_E \quad ,$$

where $G_p = 3$ V/rad.

### The other elements

The isolation amplifier is made to have a frequency response of 100 kHz with unity gain. Our analysis is limited within the frequency range of 20 kHz, hence its effect can be neglected from this analysis.

The variable gain amplifier has also good frequency response and its effect is simply described by a constant $G_T(< 1)$. In normal operation, it is adjusted as $G_T = 0.4$.

The linear-gate circuit has the gain of 0.5.

## OPEN LOOP GAIN AND ERROR REJECTION

(1) The open loop gain of the system $F(j\omega)$ is given by the multiplications of the transfer functions of the elements connected in the loop.

$$F(j\omega) = -G_0\,\frac{j\omega + \omega_2}{(j\omega+\omega_1)\left((j\omega)^2+2\eta\omega_0 j\omega+\omega_0^2\right)}$$

where

$$G_0 = \frac{\pi Q}{\Omega_0}\,\frac{\omega_1\omega_0^2}{\omega_2}\,G_c\,G_b\,G_p\,G_T \times 1.05 \times 10^4 \text{ (rad/sec)}^2 \quad ,$$

$Q = 40$, and $\Omega_0 = 2\pi \times 6 \sim 2\pi \times 8$ Mrad/sec.

We compared the above equation with the measured values in Fig. 4. The good agreement in gain curve between measured and calculated values is obtained. The difference in phase at the frequencies higher than 1 kHz is caused by the characteristics of the ferrite. The FFT processor is used in the measurement with applying white noise into the loop[3].

(2) If we express the error in resonant frequency without feedback by $\Delta\Omega(\text{off})$ and the error with feedback by $\Delta\Omega(\text{on})$, then their ratio is given by

$$\frac{\Delta\Omega(\text{on})}{\Delta\Omega(\text{off})} = \frac{1}{1 + F(j\omega)} \quad .$$

For the frequency range where $|j\omega| < \omega_{\angle}$, the above equation can be approximated by

$$\frac{\Delta\Omega(\text{on})}{\Delta\Omega(\text{off})} = \frac{\omega_0^2}{G_0}\,\frac{j\omega + \omega_1}{\omega_2}$$

The ratio has the minimum value of 1/400 at the frequencies less than $\omega_1$. It approaches to 1 at the frequency near 400 × $\omega_1$. Therefore the effective frequency range of the feedback is 400 × $\omega_1$ (i.e., ∿ 4 kHz).

## CONCLUSION

The improvements are shown in Figs. 5 and 6.

The phase errors $\phi_E$ of the three cavities in normal operation are reduced to nearly 2° in RF phase angle which is one tenth of the error existed in the old system.

The analysis is elementary, but it gives a guide in diagnosis of the auto-tuning systems.

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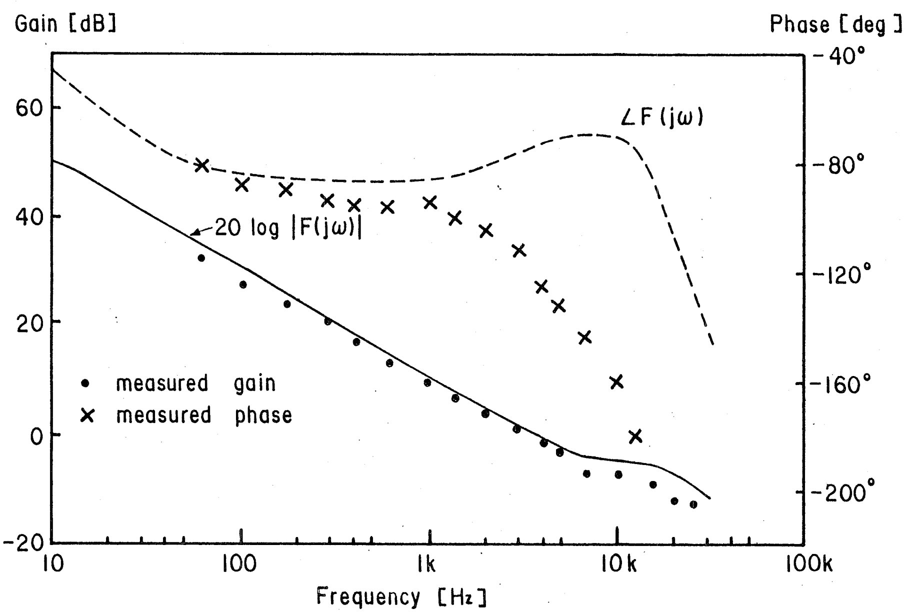
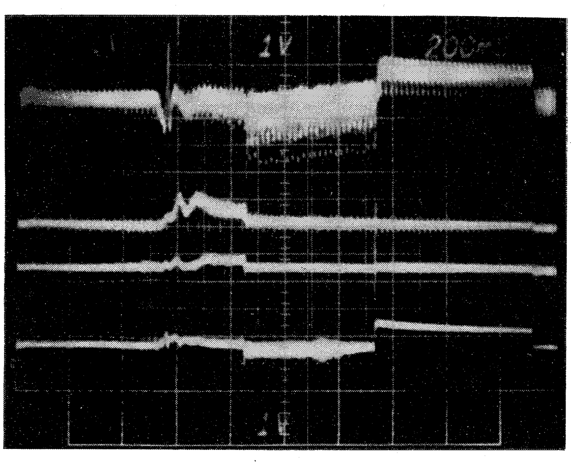



Fig. 4 The open loop gain of the system #3 at 6MHz.




Cavity Number

#0

#1

#2

#3

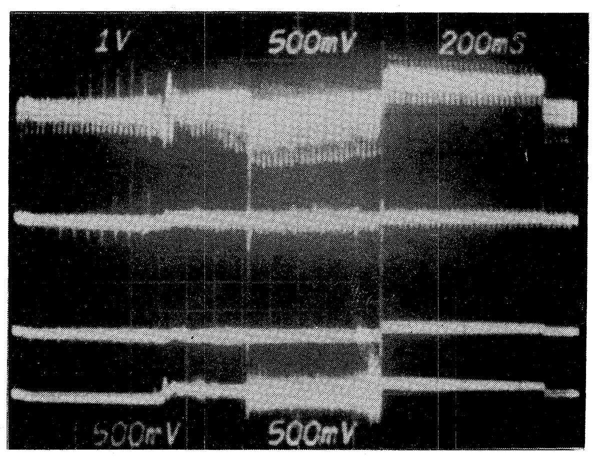



Fig. 5 $\Delta V_p$ signals of the 4 cavities. The system marked #3 is already improved.

Fig. 6 $\Delta V_p$ signals. The system marked #0 is not improved. (Note the difference in the vertical scales.)