ALIGNMENTS OF THE TRISTAN MR MAGNETS

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ABSTRACT

At the beginning of the alignment of the TRISTAN MR magnets, the provisional positions which are the bases to align the magnets, are marked on the floor of the tunnel. The relative distances between these provisional monuments are surveyed. Using the computer, the relative displacement is transformed to the radial and angular displacements of each monuments. Using these radial and angular correction values, the alignment is achieved. The procedures of the calculations to obtain the correction values and an example of the simulation are presented.

INTRODUCTION

For the TRISTAN MR magnets, the reference points (monuments) for the magnet alignment, which are determined from the geodetic survey, are marked on the floor of the tunnel. In order to correct these monuments to the desirable positions horizontally, about 400 monuments are selected out of 670 monuments in the TRISTAN MR, and at representative monuments two distances are measured. One is the distance between the adjacent monuments, and the other is the perpendicular to the straight line between the next nearest neighbor monuments. Making use of the difference between the theoretical value and measurement, the radial and angular displacements (correction values) in the horizontal plane are calculated for each monument. Thus, each monument can be corrected, and the magnets are aligned based on the corrected monuments. As a target is attached precisely on each magnet, the final magnet positions are determined by the alignment using the target, applying the same method as the monument.

The calculation of the displacements treats 800 × 800 matrix and requires the large computer memories, because the total number of monuments or targets is about 400. Therefore, some expedients are required to save the memory of the computer in the actual calculation.

EXPRESSION OF THE CORRECTION VALUES

As seen from Fig. 1, at the monument M_i, the short chord S_i (distance between the nearest neighbor monuments) and perpendicular P_i to the straight line between the next nearest neighbor monuments M_{i-1} and M_{i+1}, are expressed with R_i and Θ_i as follows[1]:

$$P_i = \frac{R_{i-1}R_i\sin\Theta_{i-1}+R_iR_{i+1}\sin\Theta_i-R_{i-1}R_{i+1}\sin(\Theta_{i-1}+\Theta_i)}{\{R^2_{i-1}+R^2_{i+1}-2R_{i-1}R_{i+1}\cos(\Theta_{i-1}+\Theta_i)\}^{1/2}} \quad (1)$$

$$S_i = (R_i^2 + R_{i+1}^2 - 2R_iR_{i+1}\cos\Theta_i)^{1/2} \quad (2)$$

$i = 1,2,----,N$ (N is the total number of monuments).

In Eqs. (1) and (2), if perpendicular P_i and short chord S_i have errors p_i and s_i respectively, these errors are transformed to the radial and angular displacements r_i and θ_i of the i-th monument. Therefore, the error p_i and s_i are described by the linearized observational equation as follows:

$$p_i = a_{i-1}r_{i-1}+a_ir_i+a_{i+1}r_{i+1}+b_{i-1}\theta_{i-1}+b_i\theta_i+b_{i+1}\theta_{i+1} \quad (3)$$

$$s_i = c_ir_i+C_{i+1}r_{i+1}+d_i\theta_i+d_{i+1}\theta_{i+1} \quad (4)$$

or in the matrix form.

$$\begin{pmatrix} p_1 \\ \vdots \\ p_i \\ \vdots \\ p_N \\ s_1 \\ \vdots \\ s_i \\ \vdots \\ s_N \end{pmatrix} = \begin{pmatrix} \cdots a_{i-1} & a_i & a_{i+1} \cdots b_{i-1} & b_i & b_{i+1} \cdots \\ & \vdots & \vdots & \vdots & \vdots \\ \cdots & c_i & c_{i+1} \cdots & d_i & d_{i+1} \cdots \end{pmatrix} \begin{pmatrix} r_1 \\ \vdots \\ r_i \\ \vdots \\ r_N \\ \theta_1 \\ \vdots \\ \theta_i \\ \vdots \\ \theta_N \end{pmatrix} \quad (5)$$

where the coefficients a, b, c and d are calculated from Eqs. (1) and (2). Solving Eq. (5), the correction values r_i and θ_i are obtained for all monuments.

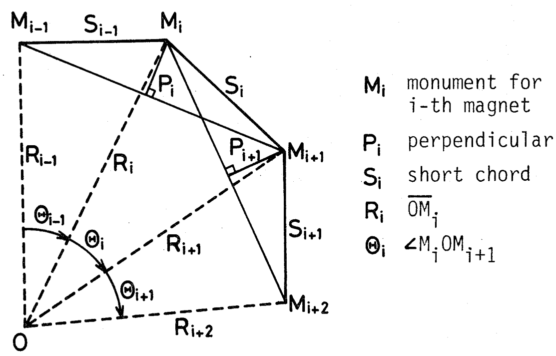


Fig. 1 Perpendicular and short chord of each monument.

PROCEDURE OF THE ACTUAL COMPUTATION

Eq. (5) is simplified as follows:

$$A\vec{x} = \vec{y} \quad (6)$$

where A is the 2N × 2N coefficients matrix, and

$$\vec{x} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_N \end{pmatrix} \qquad \vec{y} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \\ s_1 \\ s_2 \\ \vdots \\ s_N \end{pmatrix}$$

To obtain the least squares solution $\vec{x}$, Eq. (6) transforms to

$$H\vec{x} = \vec{z} \quad (7)$$

where $H = A^T \cdot A$ (2N × 2N matrix) and $\vec{z} = A^T \cdot \vec{y}$ (2N vector).

In the TRISTAN MR, the total number of the representative monuments is 396 points, so in Eq. (6) or (7) A or H is 792 × 792 square matrix, and has 627264 elements. Therefore, in the actual computation it requires some techniques to save the memory of the computer. Namely, as the matrix A has only 5 nonzero elements per column, it is reduced to 5 × 792 matrix. The other side, the matrix H is symmetric, because of $H = A^T \times A$, therefore the upper triangular elements of H are sufficient in the form of one dimensional array, and total elements of matrix H is 314028 in calculation. By the above mentioned technique,the number of elements stored in the computer is less than 26 % of the total elements of A and H.

In Eq. (7), since the matrix H has 3 degrees of freedom, the order of H must be reduced by 3. As the matrix H is symmetric, the modified Cholesky's method can be used. Consequently Eq. (7) transforms to the equation containing the coefficients of the upper triangular matrix as follows:

$$\begin{pmatrix} \diagdown & H' \\ 0 & \diagdown \end{pmatrix} \begin{pmatrix} r \\ \theta \end{pmatrix} = \vec{z}' \qquad (8)$$

Thus from the backward substitution of the Gauss's method, the solution r_i and θ_i are obtained as the radial and angular displacements of the i-th monument.

To remove the unnecessary components in the correction values (r_i and θ_i), by the parallel and rotational transformations of the coordinates and the reduction of the harmonic components, the final correction values are obtained from the following processes.

Parallel transformation

The amounts of the parallel transformation (ΔX for X-direction and ΔY for Y-direction) are given by the average of the displacement for all monuments. That is,

$$\Delta X = \frac{1}{N}\sum_{i=1}^{N}\{(R_i+r_i)\sin(\sum_{j=1}^{i-1}\theta_j+\theta_i)-X_i\}$$

$$\Delta Y = \frac{1}{N}\sum_{i=1}^{N}\{(R_i+r_i)\cos(\sum_{j=1}^{i-1}\theta_j+\theta_i)-Y_i\} \qquad (9)$$

where (X_i, Y_i) is the theoretical coordinates of i-th monument, and the angle is measured from Y-axis. After the parallel transformation of the coordinates, the correction values of each monument are obtained using the relation shown in Fig. 2 as follows:

(radial)

$$r_i' = \Big[\{(R_i+r_i)\sin(\sum_{j=1}^{i-1}\theta_j+\theta_i)-\Delta X\}^2 + \{(R_i+r_i)\cos(\sum_{j=1}^{i-1}\theta_j+\theta_i)-\Delta Y\}^2\Big]^{1/2}-R_i \qquad (10)$$

(angular)

$$\theta_i' = \sum_{j=1}^{i-1}\Theta_j' - \sum_{j=1}^{i-1}\Theta_j$$

where $$\Theta_i' = \cos^{-1}\{\frac{(R_i')^2+(R_{i+1}')^2-(S_i')^2}{2R_i'\,R_{i+1}'}\} \quad . \qquad (11)$$

Rotational transformation

The rotation angle Δϕ is given by the average of the angular correction values of all monuments as follows:

$$\Delta\phi = \frac{1}{N}\sum_{i=1}^{N}\theta_i \quad . \qquad (12)$$

After the rotation of the coordinates, the correction values are described as follows.

(radial)

The relation of the radial direction for each monument does not change by rotation, therefore

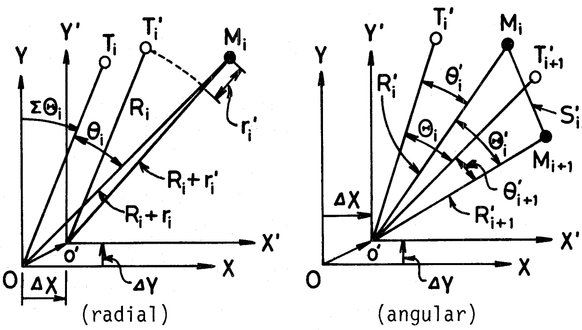
$$r_i'' = r_i' \quad . \qquad (13)$$

(angular)

The angular displacement is reduced by Δϕ, thus

$$\theta_i'' = \theta_i' - \Delta\phi \quad . \qquad (14)$$

Now the correction values are r_i'' and θ_i'' (i = 1,2,-- N).



T_i theoretical monument before transformstion

T_i' theoretical monument after transformation

M_i measured monument

X,Y coordinates before transformation

X',Y' coordinates after transformation

Fig. 2 Correction values after parallel transformation.

Reduction of harmonic components

There are introduced the irrational harmonic components in the correction values due to the measurement errors. To reduce these components the Fourier analysis is applied to both correction values r_i and θ_i independently. The harmonic components are obtained as follows:

the k-th component of the radial correction values is

$$a_k\cos(2\pi k\sum_{j=1}^{i-1}S_j/c) + b_k\sin(2\pi k\sum_{j=1}^{i-1}S_j/c) \qquad (15)$$

where

$$a_0 = \frac{1}{c}\sum_{i=1}^{N} r_i\,(\frac{S_{i-1}+S_i}{2}) \quad ,$$

$$a_k = \frac{2}{c}\sum_{i=1}^{N} r_i\cos(2\pi k\sum_{j=1}^{i-1}S_j/c)\,\frac{S_{i-1}+S_i}{2}$$

and

$$b_k = \frac{2}{c}\sum_{i=1}^{N} r_i\sin(2\pi k\sum_{j=1}^{i-1}S_j/c)\,\frac{S_{i-1}+S_i}{2}$$

with the circumference c of the TRISTAN main ring, and k = 1,2,3,----.

The low order harmonics usually have the large amplitude and a little effect to the particle motion in the accelerator even if the low order harmonic deviations are really exist, so they are eliminated mathematically without giving serious effects to the machine performance. An example of this kind is given below.

EXAMPLE OF SIMULATION

In order to estimate the accuracy of the horizontal alignment, an example of simulation is presented.

Random data are given to the correction values r_i and θ_i. Using these dummy data r_i and θ_i, p_i and s_i of each monument are calculated, which are considered to be deviations from the theoretical value. Further to reproduce the actual alignment, the measurement errors of P_i and S_i are reflected in the dummy data p_i and s_i as follows:

$$p_i' = p_i + \Delta p_i$$

$$s_i' = s_i + \Delta s_i$$

where Δp_i and Δs_i are the measurement errors and p_i' and s_i' are the final dummy data for computation. Using p_i' and s_i', the computation to obtain the correction values r_i and θ_i, is executed, and the accuracy is given by the difference between the computation results and dummy values of r_i and θ_i.

The results of the simulation under the following conditions are shown in Fig. 3 and 4.

1) standard deviation of dummy data

σ_r = 3 mm for r_i

$\sigma_\theta = 6 \times 10^{-3}$ m rad for θ_i

σ_p = 3.97 mm for p_i

σ_s = 4.16 mm for s_i

2) standard deviation of measurement error

σ_p or σ_s = 0.1 mm for Δp_i and Δs_i

Fig. 3 shows measurement errors Δp_i and Δs_i, and Fig. 4 shows the radial and azimuthal error displacements of the horizontal alignment. The result of the simulation shows that the error of the horizontal alignment is 1.1 mm rms.

REFERENCE

1) K. Endo and M. Kihara, KEK 74-3, 1974.

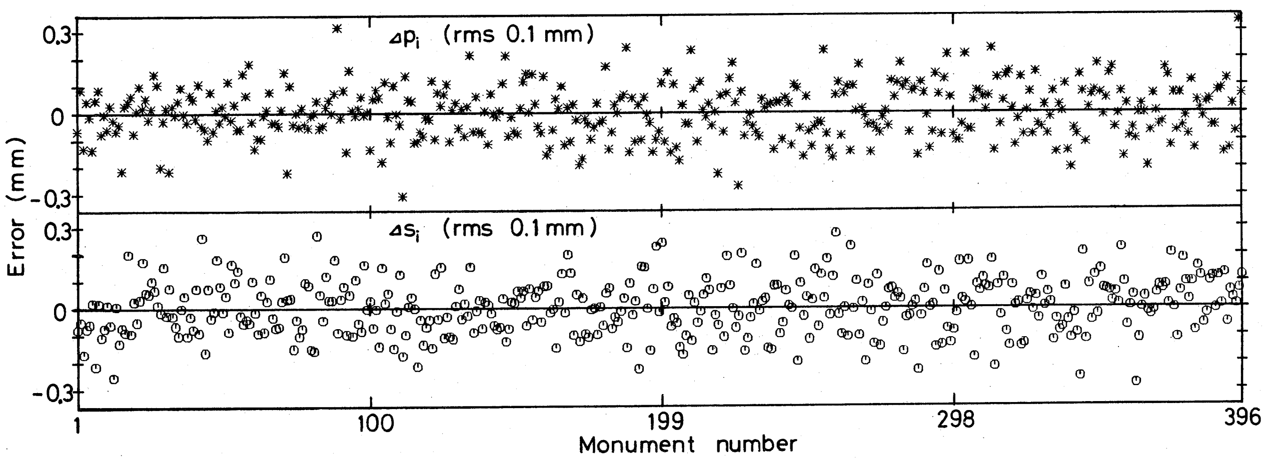


Fig. 3 Measurement errors.

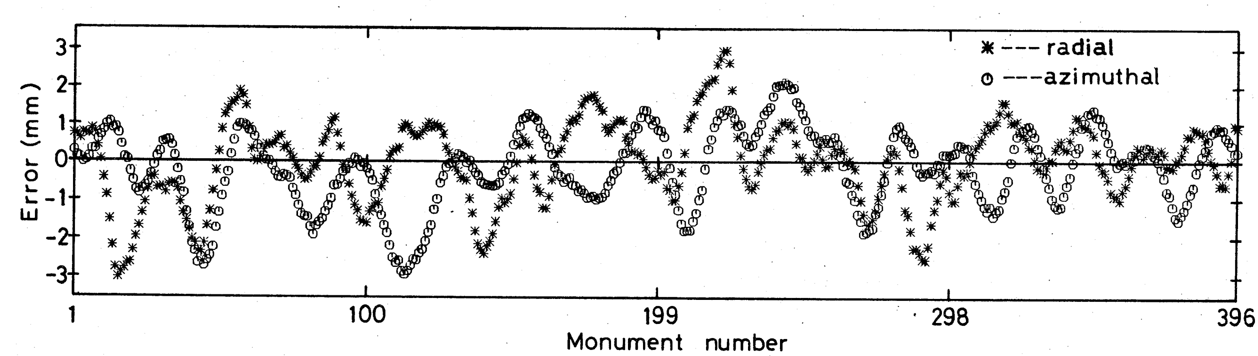


Fig. 4 Error displacements.