AN APPLICATION OF GYRO-MECHANISM TO THE ACCURATE ALIGNMENT
OF MAGNETS IN A LARGE RING ACCELERATOR

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ABSTRACT

The importance and the usefulness of azimuthal standards in the alignment process in a large ring accelerator construction are surveyed. Next, the possibility of the application of gyroscope as an azimuthal standard in the process is discussed. Finally, the error of the gyroscope in pointing the north is estimated with the design parameters of prototype gyro as an example.

INTRODUCTION

In the construction of large circular accelerator, the precise alignment of main magnets is one of the important processes. The requird accuracy of the alignment is in the range of ± 0.1 mm. Provided that the size of the accelerator is 1 km in diameter, this limit of ± 0.1 mm gives the relative error limit of 1×10^{-7}. It can be said that this accuracy is almost out of range of practical possibility from the measurement technical point of view. But fortunately, the accelerator does not require the absolute accuracy in magnet positions. The role of the magnet system is to keep the particle orbit in the vicinities of a prescribed orbit called center orbit which is supposed to go through the symmetric center of each magnet. Practically, the beam does not go around this orbit but around another orbit called closed orbit. This shift of the orbit is caused by the alignment errors and the field errors. The purpose of the precise alignment of magnet system is to reduce the discrepancies between the center orbit and the closed orbit (closed orbit distortion; COD) and to obtain a large enough useful aperture for the beam. But, if the positioning eror are quasi-uniform, i.e., the all errors are locally the same both in magnitudes and directions, the closed orbit is still on the symmetric center line of the field. In the other words, if the equivalent wave length of the position error distribution is larger than the betatron wave length, it can be said that the COD is negligiblly small. So the quasi-unfirom positon error has nothing to do with the accelerator performance. Only the local errors must be corrected. Essentially, the local errors can be reduced to the relative errors among adjacent three magnets. So the alignment of adjacent three magnets within a given error limit is the fundamental process of the overall alignment.

In an accelerator, the effect of the alignment error of magnets on the COD is not the same for all directions. The position errors perpendicular to the orbit (radial and vertical errors) are very sensitive to COD, while the one parallel with the orbit (longitudinal error) in not. Here is the key to make the alignment process very simple, and we can expect the possibility to employ the azimuthal standard.

Consider a polygon made by the center points of each (quadrupole) magnet. Assume that the vertical alignment is perfect and the each point of the polygon is on a flat plane. This assumption will be justified because the vertical alignment will be much easier than other directions. So the only algnment to be done is the radial alignment. It can be easily understood, therefore, that the local alignment in the radial direction is reduced to the adjustment of each angle of the polygon defined above. Practically, the work to be done is to adjust each magnet position radially and let each angle of the polygon agree with the design value. The alignment process can be divided into three parts; i.e., (i) the precise measurements of positions of the magnets and each angle of the polygon, (ii) data analysis and calculation by using the results of the measurement and to prepare a list of most preferable displacements for all magnets and (iii) positioning of all the magnets according to the list prepared. This process will be repeated till the errors which are given in the list as the preferable displacements are reduced within an error limit.

In the data analysis, the fact that the polygon is closed, i.e., the sum of the external angles of polygon equals to 2π, can be used as an important condition. This reduces the systematic error in the measurement. But this condition can only be used in the case of the overall alignment around the whole accelerator ring, but not in the partial alignment of the ring. In the construction of very large accelerators, it is rather usual that the installation of main magnets will be started before the completion of the accelerator enclosure as a whole. Even in such a case, if there are azimuthal standards at the both ends of partially completed tunnel, we can use a similar method to eliminate the systematic errors in measurements. The conditon is

$$\sum_{i=1}^{N} \theta_i = \Theta \qquad (1)$$

where θ_i is the i-th deflection angle between adjacent two lines which are given by connecting symmetric center points of adjacent two magents, and Θ is the total deflection angle (Fig. 1).

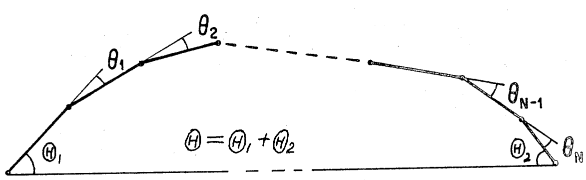

Fig. 1 A train of small angle deflections.

In principle, it may be possible to establish a new azimuthal standard in a distant place by a measurement of high accuracy. But practically, these places are in the tunnel and many steps of measurement are necessary. Therefore, to establish a new standard within an error limit of 10^{-4} or less will be almost impossible.

Gyroscope is widely used as an azimuthal standard, and is one of the most promising candidates as the azimuthal standard in the accelerator construction.

In the alignment process, measurements of magnet positions are made with an accuracy of about ± 0.1 mm, and the deflection angles is calculated by determining a thin triangle from the lengths of two sides (ℓ_1, ℓ_2) and a hight (h). Provided that ℓ_1 and ℓ_2 are about 10 m, the accuracy of the angle is about 10^{-5} rad. If the number of magnets is N, which is the number of steps of measurements, the accumulated error of the deflection angles is given as $\sqrt{N} \cdot 10^{-5}$ rad. Therefore, the accuracy of the azimuthal standard must be better than $\sqrt{N} \cdot 10^{-5}$ rad. In the case of the alignment in one quadrant of TRISTAN MR tunnel, N is about 100 and the accuracy limit of the azimuthal standard is estimated to be lower than 10^{-4} rad.

GYROSCOPE

It is said that the commercial gyroscope of highest class has an error limit of about 10^{-4} rad. and is very expensive. But this fact does not discourage us. The commercial gyroscopes are so designed as to work in a sailing or flying conditions, while the one for the magnet alignment works only in a static conditions. This will give us a possibility to realize a gyroscope with a good enough accuracy.

The gyroscope, as is well known, has a property to point the north after a some period of time adjusting its position automatically. The error in pointing the north is mainly determined by undersirable frictional forces, irregular motion of spining rotor due to the imperfections of rotor and bearing system and drift and fluctuation in rotational frequency. For the estimation of error in the angle, suppose a gyro as shown in Fig. 2. The gyro is supported by some mechanism such as gimbals and is spining with the axis roughly pointing the north.

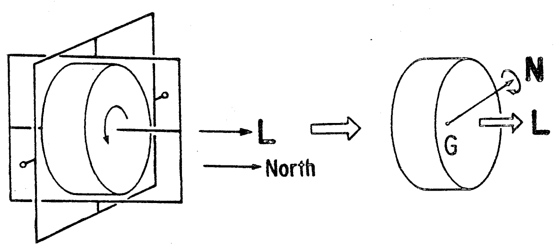

Fig. 2 A fundamental structure of gyroscope.

Many forces and torques act on the gyro, but many forces and torques are in equilibrium with the gravitational force and with the frictional torques in the bearing, and only a torque perpendicular to the rotor axis remains to be considered. This torque, $\boldsymbol{\tau}$, causes the rotor a precession which, in the design, will cancel the effect of the earth spin. $\boldsymbol{\tau}$ must fulfil the following relation, i.e.,

$$\frac{|\boldsymbol{\tau}|}{|\mathbf{L}|} = \omega_0 \sin\alpha \qquad (2)$$

where $\mathbf{L}$ is the angular momentum of the gyro. ω_0 and α are the angular frequency of the earth spin and the latitude of the place where the gyro is set, respectively. Thus, the gyro continues its motion with the axis describing a small circle around the true north. By virtue of damping force acting on the gimbals, the gyro adjusts automatically its position and finally points the true north. If the rotor is completely axially symmetric and the rotational balancing is perfect, the angular error $\Delta\theta$ in pointing the north is roughly given as

$$\Delta\theta = \frac{1}{\cos\alpha}\left(\frac{|\Delta\boldsymbol{\tau}|}{\omega_0 |\mathbf{L}|} + \frac{|\Delta\omega|}{\omega}\sin\alpha\right) \qquad (3)$$

where ω is the angular frequency of gyro and, $\Delta\boldsymbol{\tau}$ and $\Delta\omega$ are the average amplitudes of fluctuations of $\boldsymbol{\tau}$ and ω respectively.

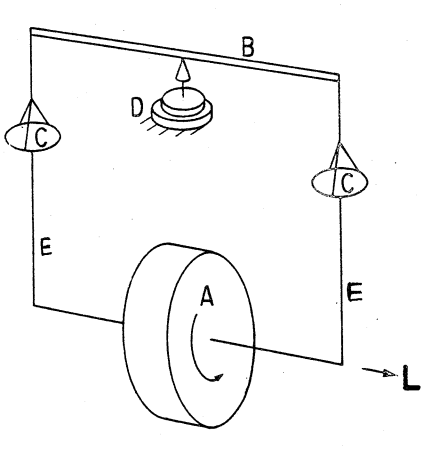

Fig. 3 A schematic diagram of prototype gyroscope.

ACCURACY OF THE GYROSCOPE

A schematic diagram of the prototype gyro which is under consideration is given in Fig. 3. A rotor (A) which is driven by a stator (not shown in Fig. 3) is suspended at the both ends by a pair of strings or rods (E). By using the knife edge and other mechanisms (D), the rotor can freely rotate around the center of gravity in any direction. To avoid undesirable forces due to stiff lead wires of power supply, the stator is supported independently of the rotor suspension system. A servo system controls the direction of the stator axis so as to make the axis parallel with the one of the rotor. The operaton of the gyro will be done as follows:

(i) The balancing of the rotor suspension will be taken in a static condition by adjusting the weights on the scale (C) of the suspension mechanism. This can be done within an error of 1 mg as in the case of a high class commercial balance.

(ii) After the static balance of the rotor has been confirmed, the rotor is set into rotation, and after the rotation reached a stationary motion, another weights are added to one of the scales which gives the rotor a torque $\boldsymbol{\tau}$. $\boldsymbol{\tau}$ must fulfil Eq. (2). If this will be done within a tolerable error, the rotor will continue to adjust its position and finally the axis of the rotor will point the true north.

In the followings, an error estimation in the above case will be done according to Eq. (3). In the first, an estimation of $\Delta\boldsymbol{\tau}$ wil be done. In this case, $\Delta\boldsymbol{\tau}$ can be divided into two parts, i.e., $\Delta\boldsymbol{\tau}_1$ and $\Delta\boldsymbol{\tau}_2$. $\Delta\boldsymbol{\tau}_1$ comes from the frictional force in the suspension mechanism of the rotor. With the fact that a typical commercial balance has a sensitivity of about 0.1 mg, we can estimate $\Delta\boldsymbol{\tau}_1$ as

$$|\Delta\boldsymbol{\tau}_1| = 10^{-4}\cdot g\cdot \ell$$

where ℓ is the arm length of the suspension mechanism (B) and g is the acceleration of gravitational force. While, the second part, $\Delta\boldsymbol{\tau}_2$, comes from a small defference between the axes of the rotor and the stator. If the angle between the two axes is ϕ (<< 1), then $\Delta\boldsymbol{\tau}_2$ can be expressed as

$$\Delta\boldsymbol{\tau}_2 = \boldsymbol{\tau}_0\cdot\phi \qquad (4)$$

where $\boldsymbol{\tau}_0$ is the driving torque acting on the rotor. If we employ a good enough positioning controller for the stator positioning, it is not so difficult to surpress ϕ down to 10^{-4} rad. or less. On the other hand, $\boldsymbol{\tau}_0$ can be extimated from the relation as

$$-I\left(\frac{d\omega}{dt}\right)_0 = |\boldsymbol{\tau}_0|$$

where I is the moment of inertia of the rotor and $\left(\frac{d\omega}{dt}\right)_0$ is the time derivative of ω at the moment of switching off of the driving power supply.

The estimation of $\Delta\omega$ will be done. If the frictional force of the bearing supporting the rotor remains constant as well as the driving power, it is not so difficult to get $\Delta\omega$ as low as $2\pi \times 0.1$ without any feed back system. So, if we employ a feed back mechanism in keeping ω constant, it will be easy to decrease $\Delta\omega$ down to $2\pi \times 10^{-2}$.

Now we are in a position to be able to estimate the accuracy of gyro, with some assumptions. For the simplicity, the following assumptions are made. The rotor is a cylinder of 20 cm in radius a and the mass M is 1 kg. The rotational frequency is 400 Hz. The damping time of the rotor in a free rotation is 100 sec, i.e.,

$$\omega/\left(\frac{d\omega}{dt}\right)_0 = 100 \text{ sec} \quad .$$

Although, this assumption is not a realistic one under the atmospheric condition, but we can operate the gyro in vacuum condition to improve the damping time. The length of arm in the balancing mechanism is 15 cm. The latitude is 40°. Under these assumptions which are summarized in the following as

$$L = |\mathbb{L}| = \frac{1}{2} Ma^2\omega = 50 \ (\mathrm{kg \cdot m^2 \cdot s^{-1}})$$

$$\ell = 0.15 \text{ m}$$

$$\alpha = 40°$$

$$\phi = 10^{-5}$$

$$\omega = 2\pi \cdot 400 = 2500$$

$$\omega_0 = 2\pi/(24 \times 3600) = 7.27 \times 10^{-5}$$

$$|\Delta \tau_1| = 0.15 \times 10^{-6} = 1.5 \times 10^{-7} \text{ N}\cdot\text{m}$$

$$|\Delta \tau_2| = |\mathbb{L}| \cdot \phi/100 = 5 \times 10^{-6} \text{ N}\cdot\text{m}$$

$$\Delta\omega = 2\pi \times 10^{-2} \quad ,$$

we estimate $\Delta\theta$ by using Eq. (3) with the result of

$$\Delta\theta = \frac{1}{\cos\alpha} \left(\frac{1.5 \times 10^{-7} + 5 \times 10^{-6}}{7.27 \times 10^{-5} \times 50} + \frac{10^{-2}}{2400} \sin\alpha \right)$$

$$= 1.8 \times 10^{-3} \text{ rad.}$$

DISCUSSIONS AND CONCLUSION

The result given in the above is a little larger than we required as the azimuthal standard. In the estimation of $\Delta\theta$, the largest contribution comes from the term of $\Delta\tau_2$. So the reduction of $\Delta\tau_2$ is most effective in reducing $\Delta\theta$. $\Delta\tau_2$ is given as

$$\Delta \tau_2 = \tau_0 \cdot \phi$$

and $\phi = 10^{-5}$ is the technical limit. So the only way to reduce $\Delta \tau_2$ is to decrease τ_0, the driving torque of the rotor. In a stationary state, τ_0 is in an equilibrium with the frictional force of the bearings supporting the rotor on both ends. In the design of the prototype gyro, any care has not been paid in reducing the frictional force. The smaller the friction in the bearing, the smaller τ_0 is. A magnetic bearing is very small in frictional force. It can be said that the magnetic bearing mechanism in the vacuum condition will be the best way to accomplish the minimum τ_0. If we succeed to reduce τ_0 by more than one order, the error of the angle $\Delta\theta$ will be reduced down to 10^{-4}, which is small enough to be used as an azimuthal standard in the accelerator construction.