A METHOD TO CONTROL $\Delta p/p$ OF THE DEBUNCHED BEAM AT KEK-PS MAIN RING

Shigeshi NINOMIYA and Hidetoshi NAKAGAWA
National Laboratory for High Energy Physics
Oho-machi, Tsukuba-gun, Ibaraki-ekn, 305, Japan

INTRODUCTION

The momentum spread of the debunched beam is varied by this method which is used in the normal operation of slow extraction mode at KEK-PS main ring. The forced oscillation of quadrupole mode is excited before debunching, and RF voltage is switched off at a phase of the quadrupole oscillation. The momentum spread of the debunched beam is determined by this phase.

A PICTURE OF QUADRUPOLE OSCILLATION

Consider the distribution of protons in $\Delta p/p$-ϕ_{rf}-phase plane. The quadrupole oscillation will not be observed when the bunch is matched with the trajectory of a synchrotron oscillation. If the axis of the phase plane are normalized such that the synchrotron motion is represented by a circle, the matched bunch occupies the area enclosed by a circle.

When the occupation of protons in the bunch is a ellipse in the normalized phase plane, the ellipse rotates around the center of the bunch with the frequency of synchrotron oscillation as shown in Fig. 1. Since we can observe the projection of the distribution onto the ϕ_{rf}-axis, the oscillation of bunch height (or width) is observed on the bunch monitor. The frequency of the observed oscillation is the twice of that of the synchrotron oscillation. This is a picture of the quadrupole oscillation.

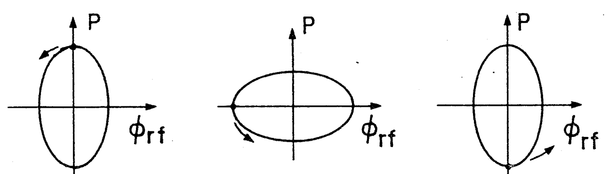

Fig. 1 The rotation of the bunch for one half period of the synchrotron oscillation.

EXCITATION OF QUADRUPOLE OSCILLATION

In order to excite the quadrupole oscillation, "phase-shake method", which was previously used for artificial blowup of the emittance during acceleration, is employed.

The method given by H.G. Hereward, which gives only qualitative behaviour of the oscillation, is used for explaining the mechanism of this method[1].

The phase equations expanded to 3rd order in $q = \phi - \phi_0$ are

$$\dot{p} = aq + b_2q^2 + b_3q^3$$
$$\dot{p} = -ap + \omega_1 \qquad (1)$$

where $p = \frac{\Delta E}{\beta}\left(\frac{-2\pi\eta h}{eVE\cos\phi_0}\right)^{1/2}$,

$$a = \frac{\omega_r}{\beta}\left(\frac{-\eta heV\cos\phi_0}{2\pi E}\right)^{1/2} \text{ (synchrotron frequency)},$$

$$b_2 = -\frac{a}{2}\tan\phi_0 ,$$

and $b_3 = -\frac{1}{6}a$.

The other notations are the same as that of Refs. 1) and 2). The synchrotron frequency a is positive below transition, negative above. In Eqs. (1), the following expansion is used.

$$\sin(\phi_0 + q) - \sin\phi \cong \cos\phi_0\left(q - \frac{q^2}{2}\tan\phi_0 - \frac{q^3}{6} + \dots\right)$$

Since the terms in Eqs. (1) containing q^2 and q^3 can be neglected for small amplitude oscillation, Eqs. (1) describe the motion whose trajectory is given by a circle. Therefore Eqs. (1) are already normalized.

In order to separate the oscillation of the center of mass (coherent part) and the relative motion (incoherent part), we divide the parameters into two parts;

$$q = \bar{q} + \delta q$$
$$p = \bar{p} + \delta p \quad .$$

Then Eqs. (1) can be divided into two sets of equations. For coherent part;

$$\dot{\bar{p}} = (a + b_2\bar{q} + b_3\bar{q}^2)\bar{q}$$
$$\dot{\bar{q}} = -a\bar{p} + \omega_1 \quad , \qquad (2)$$

and for incoherent part;

$$\delta\dot{p} = (a + 2b_2\bar{q} + 3b_3\bar{q}^2)\delta q$$
$$\delta\dot{q} = -a\,\delta p \quad , \qquad (3)$$

where terms higher than δq^2 are neglected. Note that Eqs. (2) are just the same as the phase equations (1), while Eqs. (3) describe the motion whose frequency is slightly different from that of synchrotron oscillation.

In order to describe the quadrupole oscillation, we use the following transformations.

$$z = \delta p^2 + \delta q^2$$
$$x = \delta p^2 - \delta q^2 \qquad (4)$$
$$y = 2\delta p \cdot \delta q$$

We get the following equations taking the differentiations of Eqs. (4) with respect to time, and using Eqs. (3),

$$\dot{z} = (2b_2\bar{q} + 3b_3\bar{q}^2)y$$
$$\dot{x} = (2a + 2b_2\bar{q} + 3b_3\bar{q}^2)y \qquad (5)$$
$$\dot{y} = -(2a + 2b_2\bar{q} + 3b_3\bar{q}^2)x + (2b_2\bar{q} + 3b_3\bar{q}^2)z \quad .$$

Taking the average for particles, and assuming $\dot{\bar{z}} = 0$;

$$\dot{A} = 0 \qquad (\bar{z} = A = \text{const.})$$
$$\dot{\bar{x}} = 2a\bar{y} \qquad (6)$$
$$\dot{\bar{y}} = -2a\bar{x} + 2b_2A\bar{q} + 3b_3A\bar{q}^2$$

where higher order terms in frequency are neglected since they are much smaller than 2a. Eqs. (6) describe the motion with oscillation frequency 2a, and they contain the terms of the external forces. As it is easily understood from the transformation (4), the quantity $\bar{x}$ has a large value when the bunch is high (or narrow in width), and vice versa.

The phase-shake is now described by Eqs. (6). In this case $\bar{q}$ is given by,

$$q = K\sin\omega_q t$$

where K is a constant depending upon the amplitude of phase-shake and ω_q the frequency of the shake.

In case of no acceleration where $\phi_0 = 0$ or π, therefore $b_2 = 0$, Eqs. (6) can be written as a 2nd-order differential equation;

$$\ddot{\bar{x}} + 4a^2\bar{x} = 6ab_3AK^2\sin^2\omega_q t \quad . \qquad (7)$$

The solution describing the forced oscillation is given by

$$\bar{x}(t) = -\frac{3b_3AK^2}{4(a^2-\omega_q^2)}\left\{a(\cos 2\omega_q t - \cos 2at) + 2(a^2-\omega_q^2)\sin 2at\right\} \quad . \qquad (8)$$

The resonance occurs at $\omega_q \simeq a$. If we put $\omega_q = a + \delta\omega$ and assume $2\delta\omega t \ll 1$, Eq. (8) can be rewritten as

$$\bar{x}(t) = -\frac{3}{4}b_3AK^2t\sin 2at \quad . \qquad (9)$$

During acceleration, $\bar{q}$ and $\bar{q}^2$ terms can excite the quadrupole mode. Since the effect from $\bar{q}^2$ term has already been shown, the equation containing $\bar{q}$ term must be solved;

$$\ddot{\bar{x}} + 4a^2\bar{x} = 4ab_2AK\sin\omega_q t \quad .$$

The solution describing the forced oscillation is

$$\bar{x}(t) = \frac{4ab_2AK}{(2a)^2-\omega_q^2}\left(\sin\omega_q t - \frac{\omega_q}{2a}\sin 2at\right) \quad .$$

It also resonates at $\omega_q \simeq 2a$, and is written as

$$\bar{x}(t) = b_2AKt\{\cos 2at - \sin 2at/(2a)\} \quad .$$

These results explain why the phase-shake during acceleration is effective at the frequencies of a and/or 2a.

DESCRIPTION OF THE BUNCH ROTATION

In order to calculate the momentum spread, we use the ellipse parameters α, β, and γ for describing the bunch in phase space. The bunch ellipse is given by

$$E/\pi = (q^2 + (\alpha q + \beta p)^2)/\beta \quad ,$$

where E is the area of the bunch in phase space, p and q are the same quantities used in Eqs. (1).

If the bunch height is a maximum at t = 0, the ellipse is given by

$$E/\pi = (\hat{q}_0^2/\beta_0 + \beta_0\hat{p}_0^2) \quad , \qquad (\text{with } 0 < \beta_0 < 1) \quad .$$

The integration of Eqs. (1), with $b_2 = b_3 = 0$ and $\omega_1 = 0$, gives the transfer matrix of the particle from t = 0 to t_1.

$$R = \begin{pmatrix} \cos at_1 , & -\sin at_1 \\ \sin at_1 , & \cos at_1 \end{pmatrix}$$

The transfer matrix of the ellipse parameters can be calculated from this matrix[5]. And we can get the ellipse parameters at the time t_1

$$\gamma_1 = (\cos^2 at_1/\beta_0 + \beta_0\sin^2 at_1) \quad ,$$

$$\alpha_1 = (\beta_0 - 1/\beta_0)\sin at_1\cos at_1 \quad ,$$

$$\beta_1 = (\beta_0\cos^2 at_1 + \sin^2 at_1/\beta_0) \quad .$$

The equation of β_1 gives a half width of the bunch at t_1;

$$\hat{q}_1 = \sqrt{E\beta_1/\pi} = \hat{q}_0\sqrt{\cos^2 at_1 + \sin^2 at_1/\beta_0^2} \quad , \qquad (10)$$

where $\hat{q}_0$ $(= (E\beta_0/\pi)^{1/2})$ is a half width of the bunch at t = 0.

We assume that the RF voltage is turned off at the time t_1. The transfer matrix from t_1 to t_2 ($> t_1$) is given by

$$T = \begin{pmatrix} 1 , & 0 \\ V(t_2-t_1)/p_1 , & 1 \end{pmatrix} ,$$

where $V = \Delta\omega_r R = R|\eta|\omega_r(\Delta p/p)$, R is the machine radious, ω_r the revolution frequency, and $\eta = 1/\gamma_T^2 - 1/\gamma^2$.

The ellipse parameters at the time t_2 are given by

$$\gamma_1 = (1 + \alpha_1^2)/\beta_1 \quad ,$$

$$\alpha_2 = \alpha_1 - V(1 + \alpha_1^2)(t_2 - t_1)/(p_1\beta_1) \quad ,$$

$$\beta_2 = \beta_1 - 2\alpha_1V(t_2 - t_1)/p_1 + (1 - \alpha_1^2)\{V(t_2 - t_1)/p_1\}^2/\beta_1 \quad .$$

A half width of the bunch at t_2 is given by

$$\hat{q}_2 = (E\beta_2/\pi)^{1/2} = \hat{q}_1[1 - 2\alpha_1V(t_2 - t_1)/(\beta_1p_1) + (1 + \alpha_1^2)\{V(t_2 - t_1)/(\beta_1p_1)\}^2]^{1/2} \quad .$$

Assume that $\hat{q}_2 = 2\hat{q}_1$, in other words, bunch height at t_2 is one half of the height at t_1.

$$V(t_2 - t_1)/p_1 = \alpha_1\beta_1/(1+\alpha_1^2) + \sqrt{\{\alpha_1\beta_1/(1+\alpha_1^2)\}^2 + 3\beta_1^2/(1+\alpha_1^2)} \quad .$$

Substituting V,

$$\left(\frac{\Delta p}{p}\right) = \frac{p_1\beta_1}{R|\eta|\omega_r t_{db}}\left\{\frac{\alpha_1}{1+\alpha_1^2} + \sqrt{\left(\frac{\alpha_1}{1+\alpha_1^2}\right)^2 + \frac{3}{1+\alpha_1^2}}\right\}, \qquad (11)$$

where $t_{db} = t_2 - t_1$. Since $\hat{p}_1 = (E/\pi\beta_1)^{1/2} = \hat{q}_1/\beta_1$,

$$\left(\frac{\Delta p}{p}\right) = \frac{\hat{q}_1}{R|\eta|\omega_r t_{db}}\left\{\frac{\alpha}{1+\alpha_1^2} + \sqrt{\left(\frac{\alpha}{1+\alpha_1^2}\right)^2 + \frac{3}{1+\alpha_1^2}}\right\}. \qquad (12)$$

From Eq. (12), we can calculate the value of $\Delta p/p$. If we take the value of full width of the bunch for $\hat{q}_1$, Eq. (12) gives the full width of $\Delta p/p$.

THE EXPERIMENT AND RESULTS

The setup of the equipments is shown in Fig. 2. The RF voltage of the cavity is switched off at 20 msec after P3 (the end of acceleration) pulse. We can adjust the momentum spread of the coasting beam by changing the delay time of the pulse which triggers the sine-wave generator. The examples of the forced oscillation of the quadrupole mode are shown in Figs. 3 ∿ 5. The variation in momentum spread is plotted in Fig. 6. The shaking amplitude in RF phase is nearly 20° of angle. In calculating the momentum spread, we used the measured values of $\hat{q}_1$ and t_{db} at each delay time, and used Eq. (12) with $\beta_0 = 0.5$. The quantity $\hat{q}_1$ is the value of half width measured at nearly 10 % of the bunch height.

CONCLUSION

Though the variation of the momentum spread is only from ± 0.4 % to ± 0.2 %, this is a useful method to regulate the momentum spread of the extracted beam. When we adjust $\Delta p/p = \pm 0.4$ %, the RF structures in the spill of the slow extraction are not observed.

ACKNOWLEDGEMENTS

The authoers wish to express their appreciation to Assist. Professor S. Hiramatsu for his discussions. They also express their appreciation to Professors T. Kamei and M. Kondo for their interest on this work.

REFERENCES

1) H.G. Hereward, Proc. Int. Conf. on High Energy Accelerators, 236, (1961).
2) E.D. Courant and H.S. Snyder, Ann. Phys. 3, 1, (1958).
3) D. Boussard, CERN MPS/SR/Note 70-18.
4) E.C. Raka, IEEE NS-16, No.3, 182, (1969).
5) C. Bovet et al., CERN/MPS-SI/Int (1970).

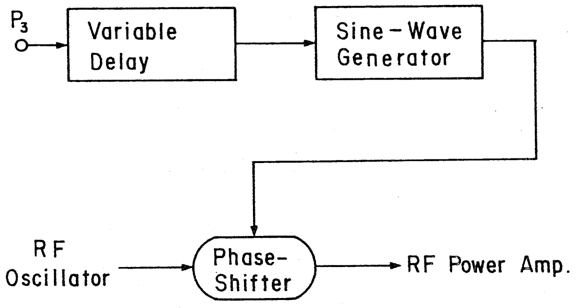

Fig. 2 The setup of the equipments.

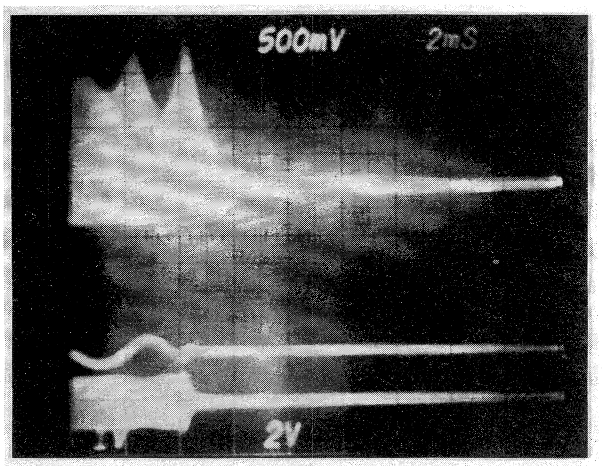

Fig. 3 The forced oscillation of the quadrupole mode. The RF voltage is switched off at the phase of a maximum $\Delta p/p$. (Delay time = 16 msec.)

1st trace; Fast intensity monitor.
2nd trace; Phase difference between the bunch and the RF voltage.
3rd trace; RF voltage.

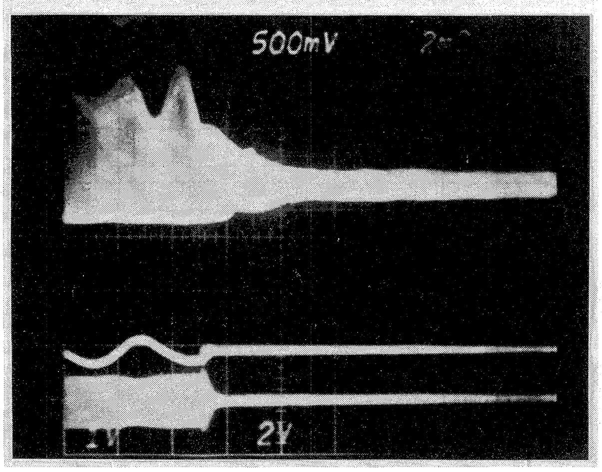

Fig. 4 Delay time = 15 msec.

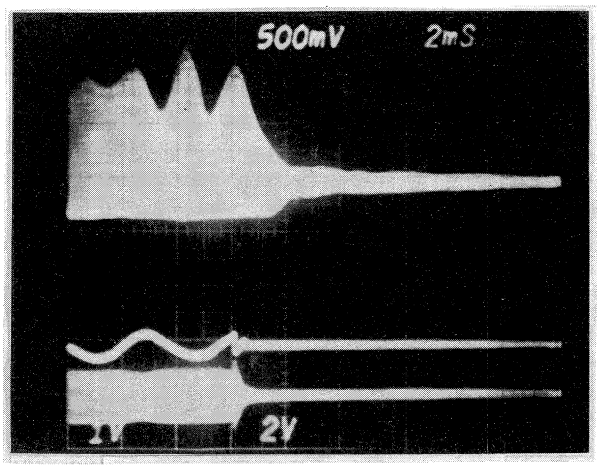

Fig. 5 Delay time = 14 msec.

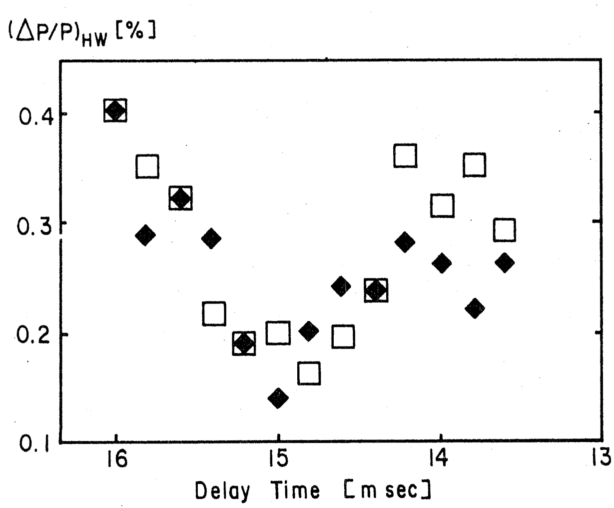

Fig. 6 The variation in momentum spread vs. delay time.