THE EFFECT OF CHROMATICITY ON INTRINSIC DEPOLARIZING RESONANCES

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Abstract

The spin tune is modulated by synchrotron oscillation. Its modulation frecquency is the synchrotron oscillation frecquency and the amplitude of modulation is proportional to $\Delta p/p$. When a beam crosses the depolarizing resonances, polarization suffers from this effect especially in the spin-flip region. The observation of this effect in the intrinsic resonances of the KEK PS and its correction by using vertical chromaticity are described.

Introduction

Polarized beams encounter depolarizing resonances during acceleration in a synchrotron. The spin of each particle precesses around the vertical axis at the frequency γG for planar rings, which is called the spin tune, where γ is the Lorentz energy factor and G is the gyromagnetic anomaly. Because of the vertical motion of the particles, the spin interacts with the horizontal perturbation fields. At a certain spin tune, the horizontal perturbation fields act on the particle spins in phase, i.e. spin precession around the horizontal axis accumulates turn by turn, then the spin which at first lies in the vertical direction is inclined gradually and finally the beam polarization decreases.

In proton synchrotrons there are two kinds of strong depolarizing resonances, the intrinsic resonance and the imperfection resonance. The intrinsic resonances are caused by the periodical focusing structure of the machine. In these resonances each particle feels a resonant field which has the same phase advance as the vertical betatron oscillation and an amplitude proportional to the vertical betatron amplitude. The imperfection resonances are due to magnet misalignment leading to vertical closed orbit distortion. All particles in the beam feel same resonant field which is periodic around the ring. Therefore these resonances occur at

$\gamma G = nN \pm \nu_z$ for the intrinsic resonances, and

$\gamma G = nN \pm k$ for the imperfection resonances.

n is an integer, N is the super periodicity of the machine, ν_z is the vertical betatron tune and k is the harmonic number of the vertical closed orbit distortion. The polarization after passage through a depolarizing resonance is given by [1)]

$$P_f = P_i(2e^{-\frac{\pi\varepsilon^2}{2\alpha}} - 1) \quad (1)$$

Here P_i and P_f are the polarization before and after passing through the resonance, ε is the resonance strength and α is crossing speed through the resonance. This means that when spin tune increases linearly in the azimuthal variable θ which is the distance along the orbit s divided by the average radius of the orbit R, polarization is maintained for the cases which are $\varepsilon^2/\alpha \gg 1$ or $\varepsilon^2/\alpha \ll 1$. Adiabatic spin-flip occurs when $\varepsilon^2/\alpha \gg 1$. Conventional resonance handling techniques are to change the ε or α to satisfy the above conditions by the fast passage method (tune jump) or closed orbit control (harmonic correction).[2,3,4)]

The beam is a collection of particles bunched by the acceleration rf as well as confined transversely by focusing fields. Each particle oscillates in energy around the synchronous energy at the synchrotron oscillation frequency. This frequency is proportional to the square root of the rf voltage and the amplitude ranges between 0 and the maximum amplitude which is proportional to the square root of the rf voltage. This oscillation causes not only a modulation of the spin tune γG but also a modulation of the resonant frequency $nN \pm \nu_z$ in the case of intrinsic resonances. This modulation leads to multiple resonance-crossing and consequently to depolarization especially in the spin-flip region, which was thought to be a drawback of adiabatic passage through depolarizing resonances. This effect was observed in the spin-flip region of the imperfection resonance $\gamma G = 2$ of SATURN 2[5)] and well explained by computer simulation[6)] and analytical methods.[7)] In the case of the KEK PS booster the estimation shows a little depolarization at the $\gamma G = 2$ caused by this spin tune modulation, can be avoided by handling the vertical closed orbit distortion. On the other hand, taking into account the spin tune modulation and the modulation of the vertical betatron tune, sizable depolarization is expected at the $\gamma G = \nu_z$ intrinsic resonance.

To avoid this depolarization due to synchrotron oscillation, the cancellation using vertical chromaticity was proposed and tested at the KEK PS. In this manuscript the brief description of the numerical calculation[8)] and experimental result will be described.

The synchrotron oscillation effect

The equation of spin motion under the magnetic fields is expressed in the particle rest frame as

$$\frac{d\Psi}{d\theta} = \frac{i}{2}\begin{pmatrix} \omega_z & \omega_s - i\,\omega_x \\ \omega_s + i\,\omega_x & -\omega_z \end{pmatrix}\Psi \quad (2)$$

$$\omega_i = \rho(s)\,\Omega_i \qquad (i = s, x, z) .$$

Ψ is a spinor and ω_z is the spin precession frequency around the vertical axis, which equals γG. The Frenet-Serret coordinate system is adopted and s, x and z express the longitudinal, horizontal and vertical components, respectively. The spin tune is

$$\gamma G = \gamma_0 G + \gamma_0 G\,\beta^2 \frac{\Delta p}{p_0} \cos(\nu_s\theta + \phi) \quad (3)$$

up to the first order of s, x, z, $\Delta p/p$. The second term of the right hand side is the spin tune modulation due to synchrotron oscillation. The element $\omega_s + i\omega_x$ expresses the rotation around the horizontal axis and can be expanded into Fourier components which have the same phase advance as the vertical betatron oscillation or vertical closed orbit distortion as follows.

$$\omega_s + i\,\omega_x = \varepsilon e^{-i\kappa_0\theta}, \quad (4.a)$$

$$\kappa_0 = nN \pm \nu_z \pm \xi_z \frac{\Delta p}{p} \cos(\nu_s\theta + \phi), \quad (4.b)$$

for the intrinsic resonances, and

$$\kappa_0 = nN + k \quad (4.c)$$

for the imperfection resonances.

where terms up to first order are taken and κ_0 is the resonance frequency. The third term of the right hand side of Eq. (4.b) is the modulation of the resonance frequency due to synchrotron oscillation.

Transforming the spinor Ψ to Φ by

$$\Psi = e^{\frac{i}{2}\sigma_z\int\omega_z d\theta}\,\Phi, \quad (5)$$

the spinor equation becomes

$$\frac{d\Phi}{d\theta} = \frac{i}{2}\begin{pmatrix} 0 & \varepsilon^* e^{-i\chi} \\ \varepsilon e^{i\chi} & 0 \end{pmatrix}\Phi \quad (6)$$

$$\chi = \int(\omega_z - \kappa_0)\,d\theta$$

Writing the elements of the spinor Φ as

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}, \tag{7}$$

ϕ_+ and ϕ_- obey the following equation:

$$\frac{d^2\phi_\pm}{d\theta^2} \mp i(\omega_z - \kappa_0)\frac{d\phi_\pm}{d\theta} + \frac{|\varepsilon|^2}{4}\phi_\pm = 0. \tag{8}$$

Polarization is expressed as

$$P = 2|\phi_+|^2 - 1. \tag{9}$$

The coefficient of the second term is

$$\omega_z - \kappa_0 = \alpha\theta + (\gamma_0 G\beta^2 \mp \xi_z)(\Delta p/p)\cos(\nu_s\theta + \delta). \tag{10}$$

During the N-th turn around the ring ($2\pi N \le \theta < 2\pi(N+1)$), $\Delta\omega_N = \omega_z - \kappa_0$ is constant and the solutions $\phi_\pm(\theta)$ are

$$\phi_+ = a_N e^{\lambda_N^+\theta} + b_N e^{\lambda_N^-\theta}$$

$$\phi_- = c_N e^{\eta_N^+\theta} + d_N e^{\eta_N^-\theta} \tag{11}$$

$$\lambda_N^\pm = \frac{i}{2}\left(-\Delta\omega_N \pm \sqrt{\Delta\omega_N^2 + |\omega_{z,N}|^2}\right)$$

$$\eta_N^\pm = \frac{i}{2}\left(\Delta\omega_N \pm \sqrt{\Delta\omega_N^2 + |\omega_{z,N}|^2}\right)$$

At the boundary between the N-th turn and N+1-th turn, we require the condition

$$\phi_\pm(2\pi N) = \phi_\pm(2\pi(N+1)).$$

Then we calculate the value of a_N, b_N, c_N, d_N iteratively with the initial conditions $\phi_+ = 1$ and $\phi_- = 0$, starting from the spin tune 20 $|\varepsilon|$ below the resonance which is sufficiently apart from the resonance and up to the spin tune 20 $|\varepsilon|$ above the resonance.

Then averaging over the transvers and longitudinal phase space, we get the beam polarization. In our calculation an uniform and quadratic distribution in phase space is assumed.

Features and corrections

From Eq.(10), this depolarization depends on the amplitude and the frequency of the synchrotron oscillation and the vertical chromaticity. Therefore the dependence of the depolarization on vertical chromaticity and acceleration rf voltage was investigated.

The calculation was performed by numerically solving the spin equation of motion. At first the behavior of one particle with various vertical emittances was calculated using $(\Delta p/p)$ and ν_s as parameters at fixed rf voltage and vertical chromaticity. Fig. 1, one of the results, shows the resonance strength vs polarization averaged over the synchrotron oscillation phase at Vrf = 12 kV and $\xi_z = -6.8$. Next this was averaged over the synchrotron oscillation amplitude and vertical emittance, using an uniform distribution in each phase space.

The calculated polarization ratios before and after the $\gamma G = \nu_z$ resonance of the booster are shown for various synchrotron oscillation amplitudes in Fig.1. The synchrotron tune, ν_s, is 1.15×10^{-3} near the bunch center. The abscissa is the resonance strength which is proportional to the betatron oscillation amplitude. The vertical emittance is between 9.8 and 17.5 πmm·mrad at resonance energy which corresponds to a resonance strength ε between 0.010 and 0.014. Displacement from the Froissart-Stora value[1] is large and causes depolarization. Averaging over the whole beam, the acceleration rf voltage dependence is shown in Fig.2. The upper and lower lines correspond to a beam emittance of 9.8 and 17.5 πmm·mrad, respectively. Depolarization is monotonically decreasing with decreasing rf voltage. At the normal operation of about 12 - 13 kV, depolarization is 20 - 40%. But less than a few percent depolarization can be achieved at 7 kV. The measured values also plotted in this figure agree well with the calculated values. This result leads to the simple correction of this depolarization; decreasing the rf voltage. This method, however, is limited by the acceptance of the rf bucket and experimentally about 9 - 10 kV is the limit for stable acceleration.

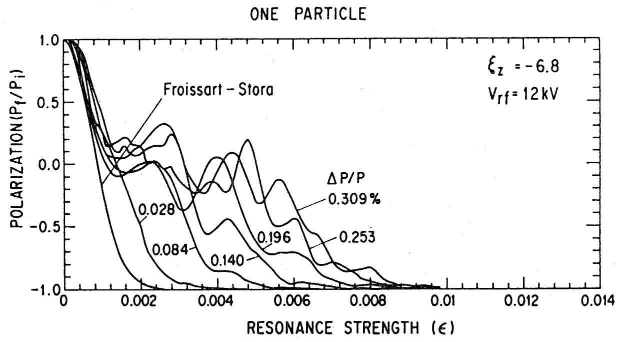

Fig. 1 Dependence of polarization on the vertical emittance averaged over the synchrotron oscillation phase at the $\gamma G = \nu_z$ resonance in the booster.

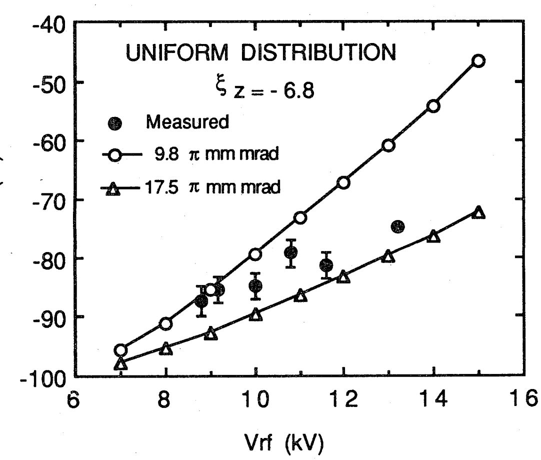

Fig. 2 Dependence of polarization ratios on the rf voltage before and after crossing the $\gamma G = \nu_z$ resonance in the booster.

If $\xi_z = \pm\gamma_0 G\beta^2$ in Eq.(10) for the resonance $\gamma G = nN \pm \nu_z$, modulation of the spin tune and of resonant frequency will cancel with each other, and multiple resonance-crossing will be avoided.[8] The result of calculation is shown in Fig.3. The significant degree of depolarization at $\xi_z = -6.8$[9] is reduced to less than a few percent at $\xi_z = \gamma G\beta^2 = 1.0$, where the remaining depolarization comes from a few percent of the non-spin-flip components at the beam center.

At first a sextupole magnet system was not available for cancellation of the synchrotron oscillation effect in the booster. Therefore, to confirm the chromaticity dependence of the polarization, a test measurement was performed at the $\gamma G = \nu_z$ resonance in the main ring.[10]

In normal operation of the polarized beam, the ramp (dB/dt) is 2.3 T/sec and rf voltage is about 40 kV. This was decreased in order to reduce the synchrotron oscillation effect.[2,3] To make the effect clear, the rf voltage was increased to 80 kV, and a slow ramp of 0.94 T/sec was adopted to get enough response from the sextupole magnet system. The result is shown with the estimated value in Fig.4. The vertical emittance is estimated to be between 3.6 and 10.0 πmm mrad at resonance energy. The polarization ratio is deduced from the continuous asymmetry data during acceleration.[3,12] A polarization ratio of about 64 % at $\xi_z = -6.5$ is improved to about 88 % at $\xi_z = 7.8$. This result clearly proves that the depolarizing effect of synchrotron oscillation at a spin-flip intrinsic resonance can be reduced by adjusting the vertical chromaticity.

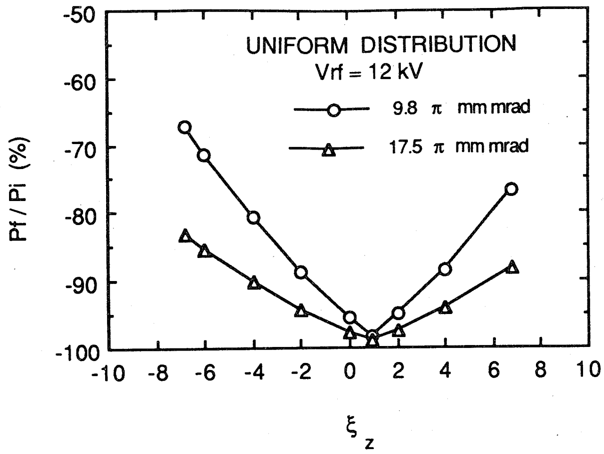

Fig. 3 Dependence of polarization ratios on the vertical chromaticity before and after crossing the $\gamma G = \nu_z$ resonance in the booster.

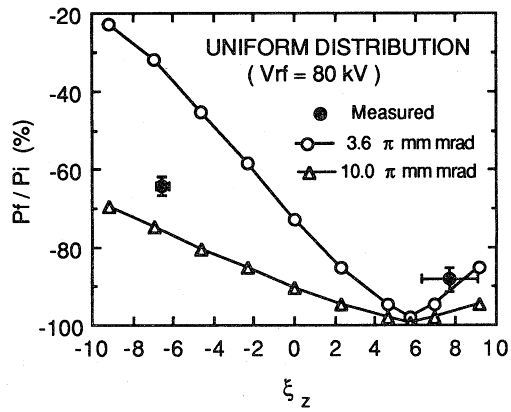

Fig. 4 Dependence of the polarization ratios on the vertical chromaticity before and after crossing the $\gamma G = \nu_z$ resonance in the main ring.

The results of the experiment [11]

Because there was qualitative agreement between the calculation and the measurement in the main ring, pulsed sextupole magnets were constructed for the booster. The vertical chromaticity can be changed by 7.8.

There are two strong depolarizing resonances in the booster during acceleration from 40 MeV to 500 MeV. One of them is the imperfection resonance, $\gamma G = 2$, at 108 MeV and another is the intrinsic resonance, $\gamma G = \nu_z$, at about 280 MeV.

At first the $\gamma G = 2$ resonance was optimized by adjusting the vertical closed orbit distortion using a vertical pulsed dipole magnet. Additional closed orbit distortion was made in order to confirm a stable spin flip, because there was a little depolarization without any additional closed orbit distortion.

The sextupole excitation was performed at two different rf voltages, 12kV and 9kV. The result is shown in Fig.5, where dependence of polarization ratios on the vertical chromaticity before and after crossing the $\gamma G = \nu_z$ resonance in the booster is poltted.

The polarization at 20 MeV before injection into the booster measured by the 20MeV polarimeter[13] was 63.66 ± 5.91 %. The polarization was improved by a relative 7.8±0.7% at Vrf =12kV and 3.3±1.1% at Vrf=9kV compared to the value at sextupole off. The maximum polarization was 54.2±0.6%.

The value of the chromaticity is calculated from the field strength because at present it is impossible to measure the vertical chromaticity in the booster. In Fig.5 the peak of the measured value is normalized to a chromaticity of 1.0 and the vertical chromaticity change was calculated. From this the vertical chromaticity will be ~0.1 without chromaticity correction. This value does not agree with the value of ~−6.8.[9] In Fig.5 the dashed line was calculated by assuming the above chromaticity ~−6.8 and a vertical emittance of 9.8 πmm·mrad. The upper and lower lines correspond to the uniform and quadratic distributions in phase space, respectively.

Beam loss was observed for sextupole currents greater than a chromaticity of about 2 by the intensity monitor. The sextupole magnets may excite the orbital resonance and this might have caused the beam loss. There remains a possibility that the orbital resonance changed the particle distribution in the transverse phase space and caused polarization loss.

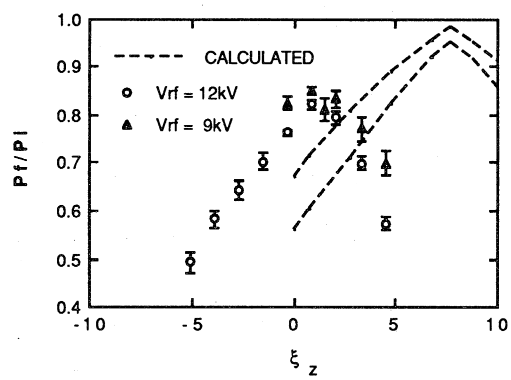

Fig. 5 Dependence of polarization ratios on the vertical chromaticity before and after crossing the $\gamma G = \nu_z$ resonance in the booster.

Conclusion

In the normal accelerating condition for the booster, i.e. at the acceleration rf voltage of 12 kV, polarization survival was improved from about 75% to about 82% by the pulsed sextupole excitation at the $\gamma G = \nu_z$ resonance. At least the sextupole field works well up to a chromaticity of about 1.0. Therefore, the measurements of the vertical chromaticity and the transverse emittance at the $\gamma G = \nu_z$ resonance are the next experiment.

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