PB08

NUMERICAL CALCULATION OF THE ELECTROMAGNETIC COUPLING STRENGTH BETWEEN THE ELECTRODES OF A BEAM-POSITION MONITOR

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Abstract

The PF 2.5-GeV linac is now being upgraded for the KEKB project. A stripline-type beam-position monitor is under development in order to easily handle the beam orbits of primary high-current electron beams for producing a sufficient number of positrons. The spatial dimensions of the mechanical monitor components were decided during its design. In the monitor design, it is particularly important to determine the opening angle of the electrode, because larger electromagnetic couplings between the electrodes generate a larger nonlinearity concerning pick-up signals. The opening angle of the electrode was decided based on a numerical calculation which considered the charge-simulation method. This report summarizes the method and results of the numerical calculation in detail.

1. Introduction

The linac is required to stably accelerate primary high-current electron beams in order to produce a sufficient number of positrons in the KEKB project[1]. The beam-position monitor (BPM) is important in order to easily handle orbits of high-current electron beams so as to suppress any beam break-up generated by a large transverse wake field. A stripline-type BPM was designed in order to perform this function. In its design, it is required to determine the mechanical spatial dimensions of the BPM components, that is, the bore radius, the stripline length, the opening angle of the electrode etc.. The opening angle of the electrode is generally designed from the point of view of the signal-to-noise ratio of the pick-up signals and the electromagnetic coupling strength between the electrodes. That is, although a large opening angle is desirable from the former point of view, it generates a larger electromagnetic coupling between the electrodes through equivalent capacitors, which are only determined by the geometrical configuration of the electrodes and the bore radius. The capacitive couplings between the electrodes do not preserve a good linearity of the pick-up voltage induced by beams, and, thus, a small opening angle of the electrode is desirable from this point of view. A numerical calculation of the electromagnetic coupling strength between the electrodes was performed in order to determine the optimum opening angle of the electrode on the basis of the charge simulation method[2]. This report describes the method and results of the numerical calculation in detail.

2. Brief overview of the charge-simulation method

A numerical analysis based on the charge-simulation method is briefly discussed here using a simple example. Two conductor rods and a ground-potential plane are arranged by some geometrical configuration (Fig. 1). The electrostatic potentials and charges on rods #1 and #2 are given as (V_1, Q_1) and (V_2, Q_2), respectively. Rod #1, #2 and the ground-potential plane are mutually electrically coupled through equivalent capacitors (C_{ij}).

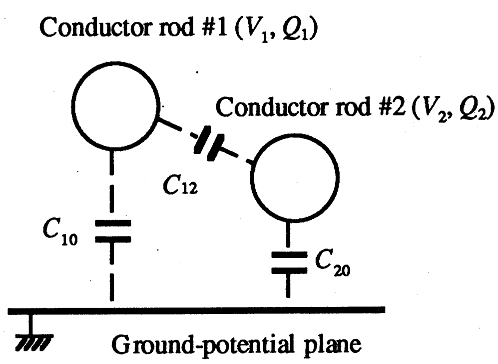

Fig. 1. Schematic drawing of a simple three-conductor system comprising two conductor rods and a ground-potential plane.

These parameters can be related by the following formula:

$$Q_1 = C_{10}V_1 + C_{12}(V_1 - V_2), \qquad (2-1)$$

$$Q_2 = C_{20}V_2 + C_{21}(V_2 - V_1), \qquad (2-2)$$

$$C_{ij} = C_{ji} \quad (i \neq j \neq 0). \qquad (2-3)$$

Here, C_{i0} and C_{ij} ($j\neq0$) are called the self-capacitance and the partial capacitance, respectively. The coupling strength between the rods is given by the ratio of the partial capacitance to the self-capacitance (C_{ij}/C_{i0}). This ratio is determined only by the geometrical configuration of the conductor system. The charge-simulation method rearranges the charge (Q_i) as a finite number of imaginary charges, q_i ($Q_i = \sum_i q_i$), on the conductor surface. The configuration of the imaginary charges is chosen so as to give an electrostatic potential (V_i) on the surface of each conductor which can be uniquely determined by the electrostatic theorem.

3. Application to the beam-position monitor

The capacitive couplings of the stripline-type BPM are shown in Fig. 2.

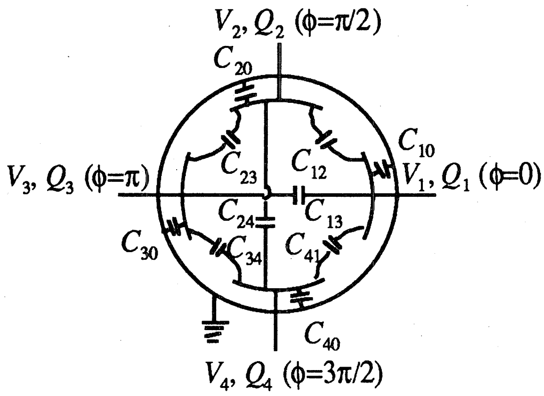

Fig. 2. Cross-sectional view of the stripline-type BPM. The equivalent capacitive couplings are also shown inside the pipe.

It is a conventional stripline-type BPM with $\pi/2$ rotational symmetry. The interval between the electrode and the inner surface of the pipe is chosen in order to make a 50Ω transmission line. The opening angle (ΔΦ) of the electrode should be determined in terms of both the signal-to-noise ratio of the pick-up signals and the coupling strength between the electrodes.

The electric field generated by relativistic beams inside the pipe can be treated as a two-dimensional electrostatic field because the field is almost boosted in the transverse direction to the beam axis. Thus, only the electrostatic field is treated in the analysis.

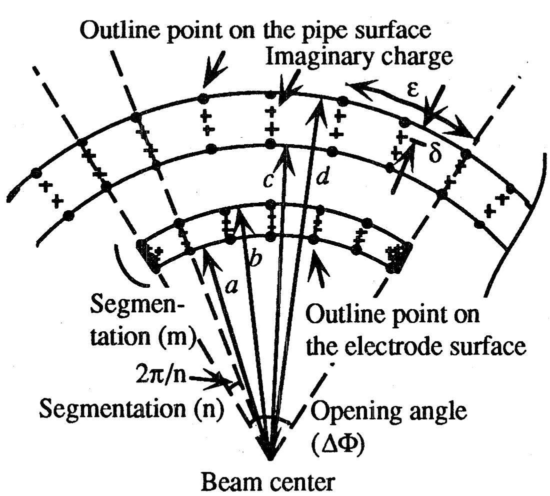

Fig. 3. Segmentation of the beam pipe and electrode surface.

First of all, the surface of the pipe and each electrode are segmented as shown in Fig. 3. The black circles indicate the outline points on the pipe and electrode surfaces, the cross points being imaginary charges. The outline points on both the inner (radius c=28.5mm) and outer (radius d=32.5mm) surface of the pipe are configured in equal (n) parts in the azimuthal direction. The electrode surface (inner radius, a=20mm; outer radius, b=21.5mm) is segmented in the azimuthal direction so as to have an azimuthal angle which is equal to the segmentation of the pipe, and is segmented in equal m parts in the radial direction. The coordinates (x_i, y_i) of the outline points are given as:

$$x_i = r\cos(2\pi i/n), \qquad (3-1)$$

$$y_i = r\sin(2\pi i/n), \quad (1 \le i \le n). \qquad (3-2)$$

The coordinates (X_i,Y_i) of the induced charges are given as:

$$X_i = (r \pm \delta)\cos(2\pi i/n), \qquad (3-3)$$

$$Y_i = (r \pm \delta)\sin(2\pi i/n), \quad (1 \le i \le n). \qquad (3-4)$$

Here, r indicates the pipe and electrode radius ($r=a+(b-a)$ j/m, ($1 \le j \le m$)) is used in the case of the electrode side surfaces), and δ is the interval between the outline point and the imaginary charge to the radial direction, which must be inside the outline point. The parameter δ is generally given by the following formula:

$$\delta = \varepsilon \times f, \qquad (3-5)$$

where ε is twice the interval length between the adjoining outline points and f is a free parameter to be tuned (to be generally chosen within 0.2~1.5). The electrostatic potential (V_j) on each outline point can be calculated by superposing the potentials generated by all of the imaginary charges, as follows:

$$V_j = \sum_i v_i, \qquad (3-6)$$

$$= \sum_i P_{ij}\, q_i, \qquad (3-7)$$

$$P_{ij} = \frac{1}{4\pi\varepsilon_0}\ln\left[\frac{(x_j - X_i)^2 + (y_j + Y_i)^2}{(x_j - X_i)^2 + (y_j - Y_i)^2}\right], \qquad (3-8)$$

where v_i is the potential generated by one imaginary charge (q_i) and P_{ij} is the potential coefficient approximated for a line charge with infinite length. The coupling strength (CS) between electrodes #1 and #2 is calculated using:

$$CS = C_{12} / C_{10}, \qquad (3-9)$$

$$= \sum_i q_i / \sum_i Q_i. \qquad (3-10)$$

Here, the summations of the imaginary charge (q_i and Q_i) are on electrode #1 and on all of the electrodes, respectively.

4. Check of the numerical calculation

The parameter f was tuned so as to produce good symmetrical and constant electrostatic potentials on the electrode surfaces. Figure 4 shows the result of a calculation in terms of the electrostatic equipotential field lines on the use of the parameter f=1, which gives the best result. The segmentation numbers (n and m) were also tuned by checking the convergence of the total induced charges on both the electrode (Q_1) and pipe surfaces ($\sum_i Q_i$). A segmentation number of n=120 (to be fixed on the segmentation number m=3) was obtained by a convergency calculation, the accuracy of which was deduced to be ~2% from the convergency. The parameters used in the calculation are summarized in the following table.

Table 1. Several parameters used in the check calculation.

Pipe potential V_0 (Volt)	0
Electrode potential V_1~V_4 (Volt)	1
Azimuthal segmentation n	60~140
Radial segmentation m	3
Free parameter f	0.2~1.5
The opening angle of the electrode (deg)	60

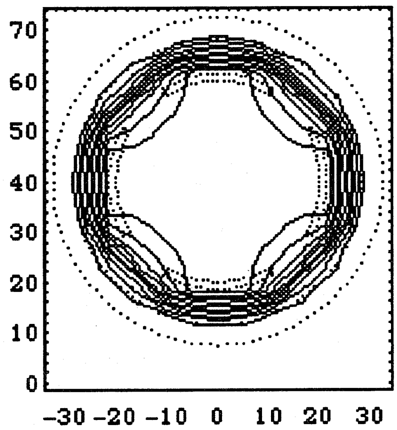

Fig. 4. Calculated electrostatic potential field lines (solid lines) inside the BPM pipe. The black points indicate the outline points of the pipe and the electrode surface.

4. Results of the numerical calculation

The coupling strength between the electrodes was obtained by using equation *(3-10)*, in which the parameters derived in the section 3 were used. Figure 5 shows the variation (solid line) of the coupling strength as a function of the opening angle of the electrode. The coupling strength is about 11% for an opening angle of 60°. The linearity of the pick-up signals is changed by the induced charges generated through the capacitive couplings. The induced charges through the couplings are approximately estimated only by taking account of the nearest-neighbor electrode using equation *(2-1)*, as follows:

$$\Delta Q_1 / Q_1 \cong \frac{C_{12}(V_1 - V_2)}{C_{10}V_1}, \qquad (4-3)$$

$$= CS\,\frac{(V_1 - V_2)}{V_1}, \qquad (4-4)$$

where ΔQ_1 is the charge induced by electrode #2. The pick-up voltage (V_i) is determined by the well-known wall-current formula:

$$V_i \propto \frac{a^2 - \lambda^2}{a^2 + \lambda^2 - 2a\lambda\cos(\phi_i - \varphi)}, \qquad (4-5)$$

where λ and φ are the displacement and azimuthal angle of the beam, and ϕ_i is the azimuthal angle of the electrode. Figure 5 shows the variations (dot lines) of the nonlinearity ($\Delta Q_1/Q_1$) as a function of the opening angle of the electrode.

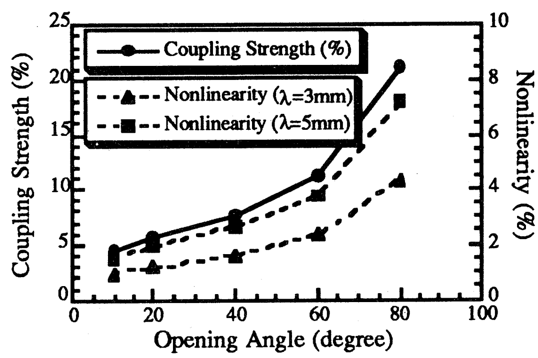

Fig. 5. Variations of the coupling strength (solid line) and nonlinearity (dot lines) as a function of the opening angle of the electrode.

5. Conclusions

The opening angle of the stripline-type BPM was analyzed on the basis of the charge simulation method. The coupling strength between the electrodes was ~11% at the opening angle 60° which generated a 2.3% nonlinearity of the pick-up signals induced by the beams.

References

[1] A Enomoto et al., "REFORMATION OF THE PF 2.5-GEV LINAC TO 8GEV", Procs. of the 17th International Linac Conference, Tsukuba, Japan, Aug. 21-26, 1994, pp.184-186; S. Kurokawa et al., "Accelerator Design of the KEK B-Factory", KEK Report 90-24, 1991.

[2] T. Kohno and T. Takuma, Numerical Calculation Method of Electric Field (CORONA PUBLISHING CO., LTD., 1980).