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High Gradient Field Generation by Multi-Harmonic Superposition

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Abstract

A method that may generate electric field with very high gradient is described. A short impulse shape wave form can be achieved by superposition of many harmonics. It is achieved by a cavity with multi-harmonic resonances, which may have ten or more modes excited simultaneously with proper phases.

1 Introduction

In some applications duty factors of beam bunches are so small that the necessary duration of the accelerating electric field are only small fraction of the repetition period of the beam[1,2]. The required wave form is an impulse train, which contains many Fourier components (see Fig. 1). It is difficult to generate such impulse in an RF cavity with high Q-value. The difficulty, however, can be overcome by superposing multiple harmonics in a cavity with harmonic higher order modes[3].

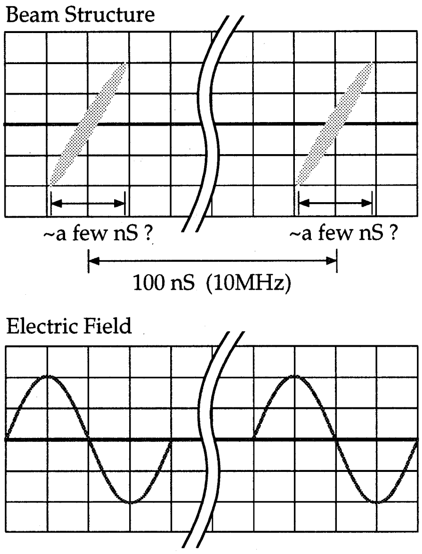

Fig. 1 Time structure of a bunched beam and electric field for rebunching (single sine wave)

2 Multi-Harmonic Impulse Cavity (MHIC)

Figure 2 shows the superposition of three harmonics of a cosine wave with amplitude of one. Assuming that every mode generates same field gradient with same order of shunt impedance, the field gradient is proportional to the number of modes. Thus the field gradient is also proportional to the total RF power, while the required power for a single mode cavity is proportional to square of the acceleration gradient. This feature may be useful for a cavity in CW operation, which has usually a limit on heat removal. Another point that should be noted is a possibility in spark limit enhancement. According to the Kilpatrick model [4], the spark limit raises with the frequency, because "the maximum possible ionic energy" decreases. For an impulse wave form, it corresponds to the reciprocal of the pulse duration, and is roughly that of the highest frequency mode applied to the cavity. The straying ions between the gaps may decrease before the next impulse, which may also help to raise the spark limit.

Figure 3 shows a single-sine-like wave form generated by superposition of three harmonics of a sine wave, which is required for the rebunching operation discussed in the former section. The peak amplitude in this case, however, is not proportional to the number of modes applied(Fig.4). This kind of wave form is useful for a double gap resonator that has two gaps with opposite signed electric field.

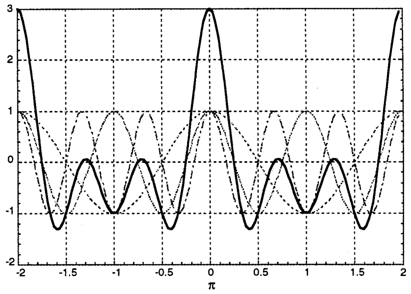

Fig.2 Superposition of three harmonics of a cosine wave with amplitude of one.

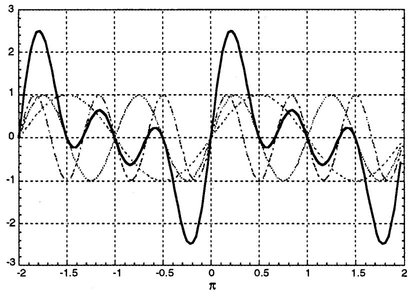

Fig.3 Superposition of three harmonics of a sine wave with amplitude of one.

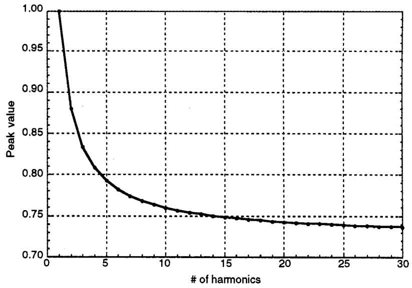

Fig.4 Peak value as a function of the number of modes.

3 Coaxial Cavity

A simple example of such resonant cavity with integral harmonics is a $\lambda/2$ coaxial cavity (see Fig. 5). Among the integral harmonics, those of even harmonics are not useful because they have nodes of electric field at the center. Assuming that the fundamental frequency is 10 MHz, a length of a $\lambda/2$ coaxial resonator becomes 15 m, which is rather huge for a construction. This size, however, may be tolerable comparing with a cylindrical cavity with such diameter. Because this structure has two gaps with different signed electric field, the cavity length should be close to $\lambda/(2n\beta)$ for high transit time factor, where $c\beta$ is the particle velocity and n is the harmonic number of the highest mode applied.

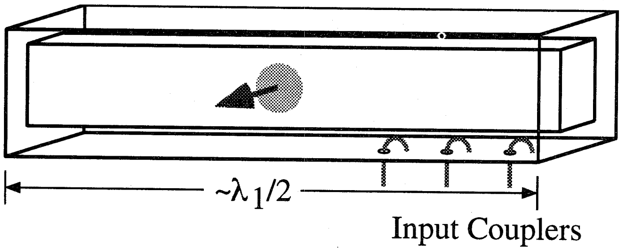

Fig. 5 $\lambda/2$ coaxial resonator with multiple input couplers.

Because there is a vacant space in the inner conductor, this space can be filled with a focusing device such as a solenoid coil or a Q magnet.

In very rough estimation assuming that shunt impedance of each mode is 4MΩ/m and 40kW/m is available for each mode, we can obtain 1MV/m for 120kW/m with three modes. There should be a lot of difficulties towards the higher number of modes. One major problem is increase of the mode density toward the higher frequency region.

In order to fine-tune the resonant frequencies, many frequency tuners are desired. An RF power feeding scheme should be established. Although a cavity with single RF coupler is simple, the RF circuit may be complicated because many harmonics have to be combined and matched to the cavity. On the other hand, installing multiple couplers on a cavity makes the fabrication complicated. Figure 6 shows a schematic drawing of a proof of principle (POP) model for a coaxial cavity geometry (CoaxPOP). Coupling scheme, frequency tuners and RF characteristics will be investigated on this cavity. Experiments on the discharge properties are also possible so that it is designed as vacuum tight. Four of frequency tuners and coupling loops are prepared (see Photo 1). One antenna can be installed at the center port for measuring the electric field.

Figure 7 shows a preliminary data from the CoaxPOP, where RF power transmission is measured between a) both end loops and b) from end loop to center antenna. The fundamental frequency agrees with the designed value of 144.5 MHz. Only harmonic modes are observed up to seventh harmonics. Even harmonics are much suppressed at the center antenna as expected. The measure frequency, Q-value and the calculated values are listed on Table 1. For the

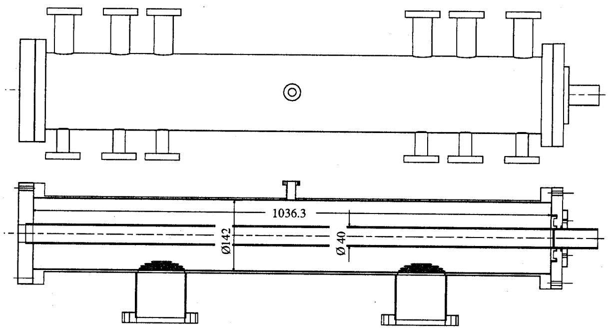

Figure 6: Proof of principle model for coaxial cavity.

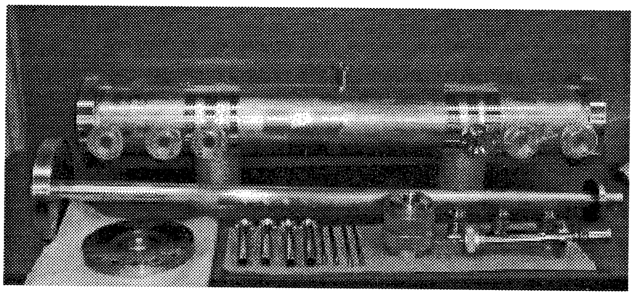

Photo 1: CoaxPOP parts

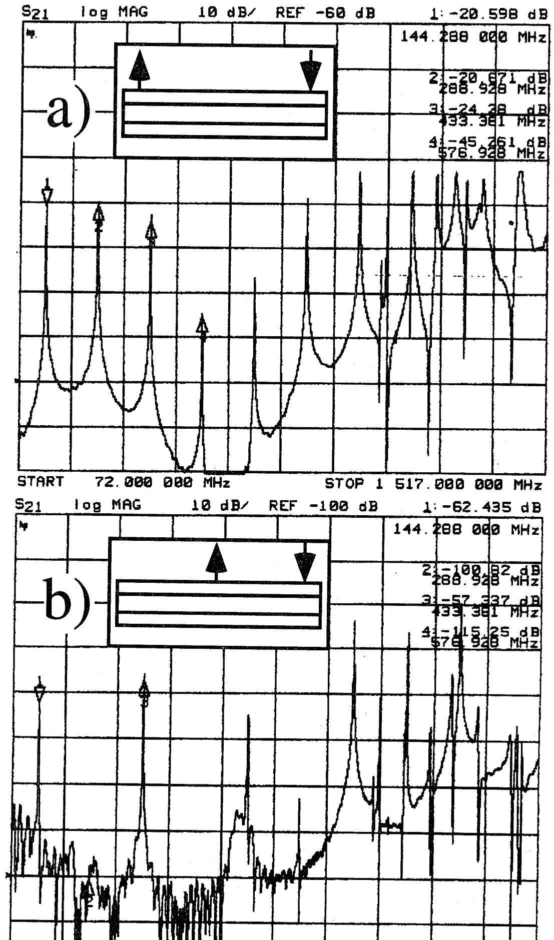

Figure 7: Measured transmissions between a) both end loops and b) from end loop to center antenna.

fundamental mode, the shunt impedance is estimated as 1MΩ and it can produce 300kV with 100kW input power between outer conductor and inner conductor at the center. The surface electric field on the inner conductor will be 12MV/m at the center, while the Kilpatrick criterion gives the same level. Because the maximum electric field will exceed the criterion by adding the same level of higher harmonics, the spark limit increase may be check by this procedure, if amplifiers are available. Further measurements are under way.

Table 1
Measured and calculated values for CoaxPOP

Frequency	Q(meas.)	Q(cal.)	Z [MΩ](cal.)
144.5	5700	6700	1.3
433.5	9200	11600	0.75
722.5	10000	15000	0.58

3 Cylindrical Cavity

Because a λ/2 coaxial resonator has two acceleration gaps, it has restrictions on designing a system as stated above. A single gap cavity using multiple modes such as "signal-cavity with double frequency buncher" [5] is preferred in some applications. Figure 8 shows the mode spectra of TMmn0 in a simple cylindrical cavity. Unlike the coaxial cavity, frequencies of higher order modes in a cylindrical cavity are not integral multiple of that of a fundamental mode. Adjustments are needed to satisfy such requirements.

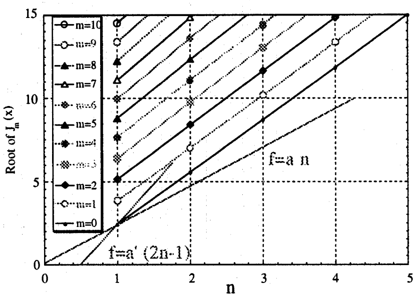

Fig. 8 Mode spectra of TMmn0 in a simple cylindrical cavity.

4 Offsetting Multi-Harmonic Impulse Cavity (OMHIC)

Although radially folded cylindrical cavities[3] are adequate for some applications because of the smaller cavity radius, it is not easy to adjust the frequencies to multiples of the fundamental frequencies. The frequencies of a cylindrical cavity, however, are almost on a line that does not cross the origin. Fitting five frequencies by a line, the frequency $F(n)$ normalized by the frequency separation is approximated by,

$$F(n)=n-0.235. \qquad (1)$$

It seems easier to modify the constant 0.235 to a reciprocal of an integer (e.g. 1/4 or 1/5) than to modify the mode frequencies to multiples of the fundamental one. If it is possible to modify the constant to 1/4, the system that is a superposition of five modes is

$$f(t)=\sum_{n=1}^{5} cos\left(n-\frac{1}{4}\right)t. \qquad (2)$$

In this scheme, the lowest mode has angular frequency ω of 0.75. Figure 9 shows the wave form superposing up to five modes. This scheme is thought to be a partial set of the harmonics series missing many harmonics including fundamental one(ω=1/4). If we rescale the frequency four times, the included frequency is ω=3,7,11,15,19,... (4i-1). This Offsetting Multi-Harmonic Impulse Cavity (OMHIC) still has periodical high peaks, and may be practical compared with the MHIC, although the repetition cycle becomes long and extra pulses appears.

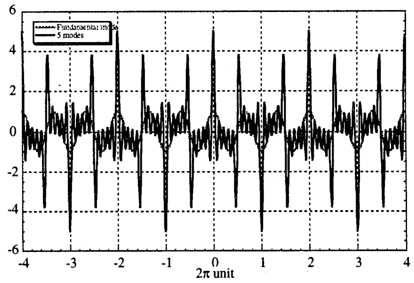

Fig. 9 OMHIC wave form with offset of 1/4 and the number of modes of five.

5 Multi-Higher Order Mode Impulse Cavity (MHOMIC)

For a very low duty factor application, even the periodicity is not needed. If the cavity has filling time of short enough to work in pulse operation and only one short bunch comes in one pulse, the phase of each mode needs to coincide at one moment. This scheme can utilize any modes that can accelerate beams, which seems more flexible that previous scheme when the duty factor is very small.

Because these schemes need power for multiple frequencies, power feeding scheme should be established to realize them.

References

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