A Study on a High Performance Quadrupole Magnet for an Accelerator

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Abstract

A theoretical method using three dimensional Laplace's equation for determination of the pole shape of a quadrupole magnet was studied. The calculation shows some distinctive features, one of which is a shim structure. Three dimensional numerical calculation was also carried out. Comparing the numerical result to the preliminary experimental result shows that the method may be very useful to determine the pole structure reasonably.

1 Introduction

In recent years, many synchrotron-type accelerators have been constructed not only for physics but also for technology and medical care. Under this condition, cost performance of a synchrotron becomes required. One of the answers is a smaller synchrotron, which requires accelerator elements to develop qualitatively.

In a quadrupole magnet case, a thin quadrupole magnet is strongly required for a small synchrotron. This request is, however, not so easy because an edge effect to a quadrupole field must be considered when a quadrupole magnet is designed. In other words, usual way to determine the shape of the magnetic pole of a quadrupole magnet, i.e., empirical ways or two dimensional computer calculations must be not enough in the small-type quadrupole magnet. Therefore, as a start point of a magnetic pole shape, the solution of three dimensional Laplace's equation should be adopted[1]. Although the final shape of the magnetic pole may be a little bit changed because both of presence of iron and of magnetic hysteresis, it should be possible to determine the pole shape experimentally.

2 Three dimensional Laplace's equation

The magnetic scalar potential, ϕ, for a quadrupole magnet is assumed as[1]

$$\phi(r,\theta,z) = -\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} r^n b_m \sin(m\theta) g_n(z). \qquad (1)$$

Since the potential, ϕ, should be solution of three dimensional Laplace's equation, i.e., $\triangle\phi = 0$, we got the following relationship,

$$g_n(z) = -\frac{g''_{n-2}(z)}{(n+2)(n-2)} \quad (n \geq 3). \qquad (2)$$

The equation to represent the pole shape of a quadrupole magnet is

$$\frac{1}{\sin(2\theta)} = -2\sum_{\mu=1}^{\infty} k(\mu) g_2^{(2\mu-2)}(z)\frac{r^{2\mu}}{R^2}, \qquad (3)$$

where R is a bore radius at the center of the quadrupole magnet and

$$k(\mu) = \frac{(-1)^\mu}{4^{\mu-1}(\mu+1)!(\mu-1)!}. \qquad (4)$$

The function $g_2(z)$ was adopted as

$$g_2(z) = \frac{1}{2}\frac{erf(\frac{z+z_0}{\sigma}) - erf(\frac{z-z_0}{\sigma})}{erf(z_0/\sigma)}, \qquad (5)$$

where $2z_0$ is a measure of the pole length and σ is a measure of the stray magnetic field extension. The example of $g_2(z)$ is shown in fig.1.

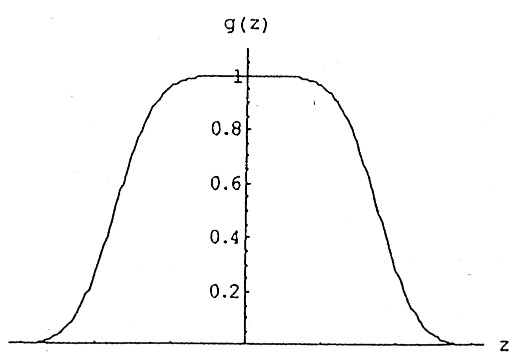

Figure 1: A typical shape of the function $g_2(z)$.

Though right hand of eq.(3) includes infinite terms, we calculate it mathematically until $\mu = 5$, which is shown in fig. 2. It is seen that the outline of the pole shape (1)is similar to hyperbolic curve at the center, (2)has some bulge compared to the hyperbolic curve in the intermediate region and (3)is quite different from the hyperbolic curve near the pole edge. It should be noted that the calculated pole shape is similar to those of present quadrupole magnets. Especially, the bulge in the intermediate region may correspond to a well-known shim structure. An end shim structure is also seen to be predicted in this calculation.

3 Numerical calculation

In order to ensure this conceptual idea, a normal-type quadrupole magnet (i.e. "thick" magnet) with widely adjustable edge piece was prepared. Figure 3 shows schematic drawing of the quadrupole magnet. The length of the adjustable edge piece is 40 mm.

Before experiments three dimensional magnetic field computational calculation by the TOSCA code[2] which

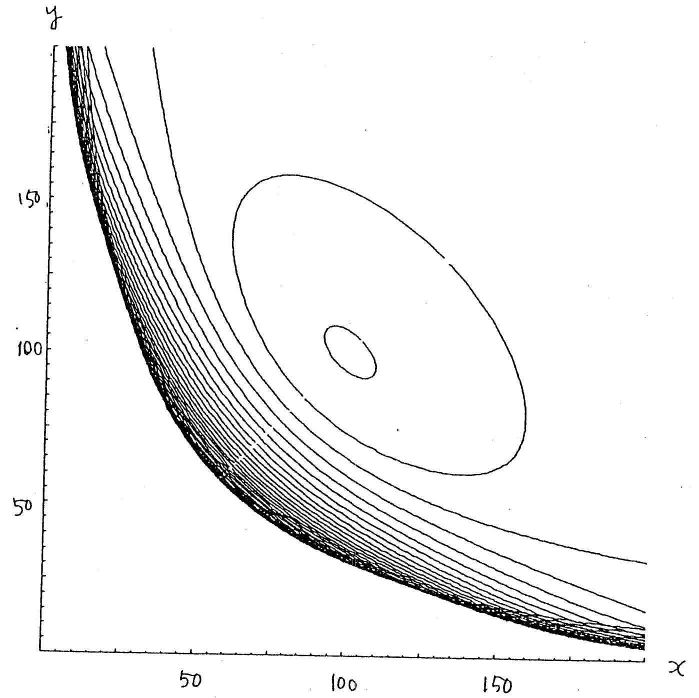

Figure 2: A numerical calculation of the eq.(3), but until μ=5.

includes an iron effect and a magnetic hysteresis effect was carried out for two cases, i.e., (1)no further manufactuaring of the edge pieces(Modell) and (2)manufactuaring the pole end based on eq.(3)(Model2). Figure 4 shows calculated "integral GL-product", $\frac{\Delta GL}{GL_0}$ which is defined in this report by

$$(\frac{\Delta GL}{GL_0})(x) = \frac{\int_{-\infty}^{\infty} G_x(x,z)dz - \int_{-\infty}^{\infty} G_0(0,z)dz}{\int_{-\infty}^{\infty} G_0(0,z)dz}, \quad (6)$$

where

$$G_x(x,z) = \frac{\partial B_y(x,z)}{\partial x}. \quad (7)$$

In this calculation, Model2 is seemed to be worth than Modell, which, however, may come from the limitaion of the calculation. Comparing the experimental result discussed below, only the ratio of tha value in Model2 to that in Modell seems to be important.

4 Preliminary experimental results

Some preliminary integrated GL-product measurements were carried out with no change of the adjustable edge pieces, which is shown in fig. 5. Experimental results are quite different from the calculations by the TOSCA code shown in fig. 4.

We assumed that the three dimensional numerical calculations of the magnetic field are inadequate in the absolute value and, meanwhile, are possible to treat to be useful in the relative value within some limitations. Thus, one can estimate the integral-GL product with manufactuaring the pole end based on eq.(3), $\Delta GL_{est}/GL_0(x)$,

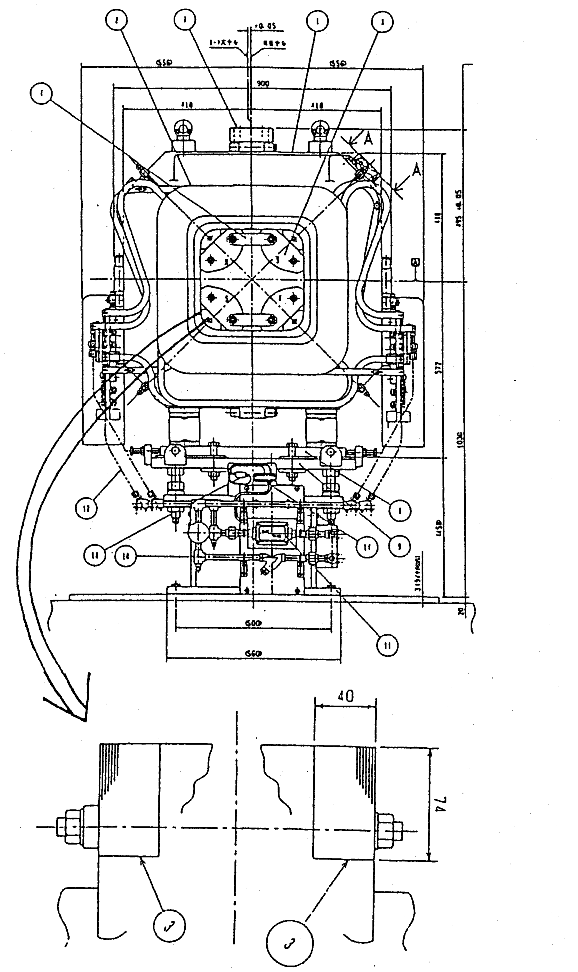

Figure 3: A schematic drawing of the quadrupole magnet.

from that without manufactuaring of the edge pieces which is experimentally obtained, as

$$(\frac{\Delta GL_{est}}{GL_0})(x) = (\frac{\Delta GL_{exp}}{GL_0})(x) \times \frac{(\frac{\Delta GL_{model2}}{GL_0})(x)}{(\frac{\Delta GL_{model1}}{GL_0})(x)}, \quad (8)$$

which is also shown in fig. 5.

It was found out that the estimated values predict that the quadrupole magnet with manufactuaring the pole end based on eq.(3) has a wider good magnetic field region, which is in accord with the conceptual idea. Though the physical base of the asssumption described above is rather weak, one may hope that the analytically calculated pole structure by using the three dimensional Laplace's equation is useful.

Now the edge pieces of the magnet with the shape based on the analytical solution of the three dimensional Laplace's equation have been made and the magnetic field measurements are under way. In the very preliminary measurements, the good magnetic field region is seen to be widen, which is expected from this conceptual idea.

References

[1] B. Langenbeck, IEEE Transcation on Magnet **24** (1988) 1369.

[2] Vector Field Ltd., "OPERA-3d REFERENCE MANUAL", VF-09-96-D4

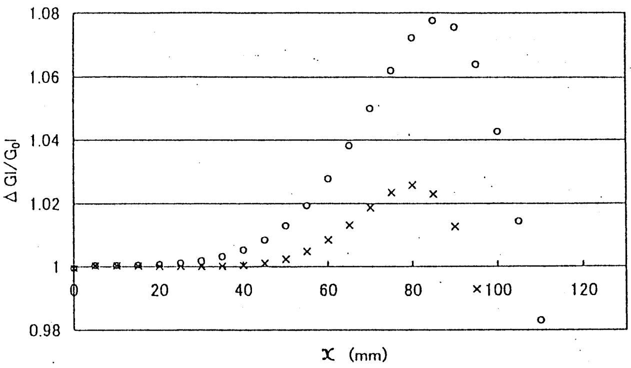

Figure 4: $\Delta GL_{model1}/GL_0$(x) (cross) and $\Delta GL_{model2}/GL_0$(x) (circle) as a function of x.

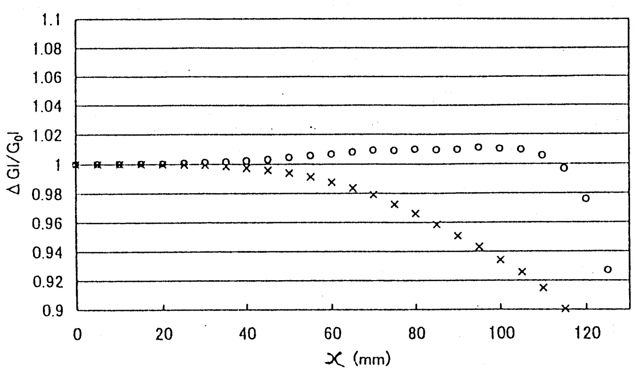

Figure 5: $\Delta GL_{exp}/GL_0$(x) (cross) and $\Delta GL_{est}/GL_0$(x) (circle) as a function of x.