Bunch Shaping by RF Voltage Modulation with a Band-limited White Signal

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Abstract

In the 12-GeV synchrotron of the KEK PS, beam losses during the injection flat bottom and at the transition energy have been a serious and final obstacles to beam intensity upgrade for the long-baseline neutrino-oscillation experiment. To supplement existing schemes to cope with the losses, an additional method has been proposed: uniform bunch formation by modulating acceleration rf voltage with a band-limited white signal. A unique feature of this method is to increase the bunching factor without the longitudinal emittance blow-up, in principle. The temporal evolution of the beam distribution and the time constant have been evaluated through computer simulations and analytic considerations. This method has been successfully applied to the KEK PS. The intensity goal of 6×10^{12} protons per pulse has been achieved since May, 1999.

1 Introduction

The beam loss during the injection flat bottom resulted from space charge force related phenomena and the other loss at the transition energy (γ_t) resulted from a temporal defect in the current low-level rf feedback system or a microwave instability remain until recently, in spite of all our efforts [1][2][3]. Because of these losses, we could not reach the intensity goal of 6×10^{12} protons per pulse (ppp) at the beginning stage of the commissioning.

On the other hand, it has been well known that mitigation of the local beam density is quite effective to avoid any collective instabilities [4][5]. In the Main Ring a method so called "$2f_s$ anti-damping" was used [6], in which the quadrupole oscillation mode in the longitudinal motion is enhanced by feeding the bunch signal onto the accelerating voltage. This method was effective in some extent. However the performance was not very reliable because it resorts to positive feedback. In this context bunch shaping by rf voltage modulation with a band-limited white signal has been proposed [7].

We applied this method to the moving buckets in the 500 MeV fast-cycling synchrotron (Booster) and the 12 GeV synchrotron (Main Ring), and successfully achieved the intensity goal of 6×10^{12} ppp.

In this paper we describe the theory of the method and report the experimental results applied for stationary buckets and moving buckets in the Booster and Main Ring.

2 Theory

Fundamental rf voltage modulation with band-limited white signal was proposed in order to manipulate the bunch without longitudinal emittance blow-up.

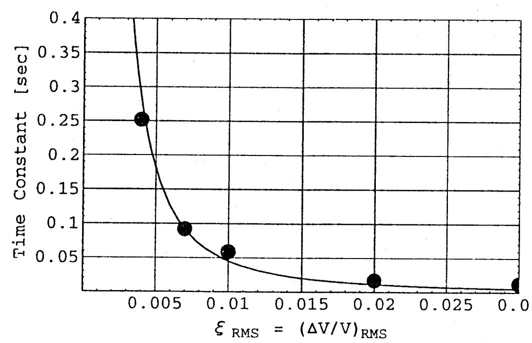

Fig. 1 Time constant as a function of the degree of modulation.

The particles trapped in the rf bucket oscillate with different synchrotron frequencies, because of the rf potential nonlinearity. Assume that we apply the rf voltage modulation with a band-limited white signal: the frequency band extends from $2\omega_0 - \Delta\omega$ to $2\omega_0$ (ω_0: synchrotron angular frequency of zero synchrotron amplitude and $\Delta\omega$: bandwidth). The particle located within the frequency spectrum, resonates in one of the band of the frequency spectrum and changes its amplitude. Then, the particle changes in synchrotron frequency due to a nonlinear rf potential, resonates at another frequency and continues a random walk in the longitudinal phase space. However, this continues only until reaching the edge of the band of the frequency spectrum. The amplitude cannot increase any further because there is no driving force of the resonance. Consequently, particles diffuse in a bounded region and form a uniform distribution in the longitudinal phase space.

The temporal evolution of the distribution was analytically derived for a stationary bucket by using diffusion equation:

$$\frac{\partial\rho(J,t)}{\partial t} = \frac{\partial}{\partial J}\left(\frac{A_2}{2}\frac{\partial\rho(J,t)}{\partial J}\right), \tag{1}$$

with

$$A_2 \approx \begin{cases} \pi\omega_0^2\Lambda_0 J^2 & if \quad 0 \le J \le J_1, \\ 0 & if \quad J_1 < J, \end{cases}$$
$$J_1 \approx 8\pi\Delta\omega, \tag{2}$$

where J is the action variable, $\rho(J,t)\Delta J$ is the number of particles found between J and $J+\Delta J$, $A_2/2$ is a diffusion constant which is determined by the modulation dynamics and spectrum, and Λ_0 is the power spectrum density of the amplitude modulation $\xi(t)$. The temporal evolution of the distribution $\rho(J,t)$ is derived in [7]. And then the time constants τ of $\sqrt{<\phi^2>}$ and

$\sqrt{< (\Delta p/p)^2 >}$ are roughly estimated as

$$\tau \sim \frac{4\Delta\omega}{\pi\omega_0^2\xi_{RMS}^2},$$

where $\xi_{\rm RMS}$ is the root mean square of $\xi(t)$.

The numerical simulation was also performed using N equaly spaced spectral lines [7]. The energy gain of the nth turn is expressed as

$$\Delta E_{n+1} - \Delta E_n \quad = \quad eV(1+\xi_n)\sin(\phi_n),$$

where ΔE_n and ϕ_n are the energy deviation from the synchronous particle and the phase of the particle with respect to the rf voltage at the nth turn, respectively, e is the electric charge, V is the unperturbed rf voltage and ξ_n expresses the stochastic modulation at the nth turn:

$$\xi_n = \xi_{\rm RMS}\sqrt{\frac{2}{N}}\sum_{j=0}^{N-1}\sin[n(2\omega_0 - j\frac{\Delta\omega}{N-1})T_{\rm rev} + \psi_j],$$

with ψ_j, an initial phase, which is determined by a random number between 0 and 2π and $T_{\rm rev}$, the revolution time. In the simulation N is taken as 100.

Following both approaches, the time constants was evaluated with typical parameters at the Main Ring injection energy as plotted in Fig. 1. They are plotted as a function of the rms degree of modulation ($\xi_{\rm RMS}$). Solid line indicates the analytical calculation and dots are the simulation results. The results of both calculations agree very well and suggest that time constants are short enough for practical application.

The limit of this method comes from higher order parametric resonances. In this method we make use of the parametric resonances $2f_s = f_{mod}$. However, due to nonlinearity of the rf potential, higher order parametric resonances [8], $2nf_s = f_{mod}$, take place in principle, where f_s is the synchrotron frequency of any amplitude and f_{mod} is the frequency of modulation. To avoid such higher order effects, we should confine the synchrotron frequency spread within f_0, i.e. $f_0 \le f_s \le 2f_0$. Fortunately this corresponds to safety phase region of $-160° \le \phi \le 160°$, wide enough for usual operation.

In the case of moving buckets the condition is more restrictive because of asymmetric rf potential.

3 Experiments

The test was performed during the injection flat bottom of the main ring on June 11, 1998 and in successive experiments. Voltage modulation has been easily implemented in the existing rf system. The pseudo band-limited white signal which consists of many spectral lines was generated by the 2 MS/s arbitrary waveform generator (Pragmatic 2711A). The twice of the synchrotron frequency was 11.6 kHz, total spectral lines ~130 and the frequency bandwidth of 2.65 kHz. This signal was fed into the voltage control loop in the low-level rf feedback system, which has a frequency band width of ~30 kHz [6]. The typical frequency spectrum of the rf voltage with modulation was observed as shown in Fig. 2. The left peak corresponds to the rf frequency, the flat-topped spectrum on the right side is the side-band due to the rf voltage modulation and the small peak at the center is just the marker. For the degree of modulation of ~3.6%, a pseudo-uniform distribution was successfully obtained within a few tens ms.

The bunch shapes without/with modulation at 48 ms after modulation start are shown in Figs 3 and 4. In order to evaluate the phase space density, we make use of superposition of uniform distributions ($f_{\rm k}(r)$) of different radius ($r_{\rm k}$) in the longitudinal phase space:

$$f(r) \quad = \quad \sum_{\rm k} a_{\rm k} f_{\rm k}(r),$$

$$f_{\rm k}(r) \quad = \quad \begin{cases} 1 & if \quad 0 \le r \le r_{\rm k}, \\ 0 & if \quad r_{\rm k} < r. \end{cases}$$

The projection on the time axis is $\phi(t) = \sum_{\rm k} a_{\rm k}\phi_{\rm k}(t)$, where

$$\phi_{\rm k}(t) = \begin{cases} 2\sqrt{r_{\rm k}^2 - t^2} & if \quad |t| \le r_{\rm k}, \\ 0 & if \quad r_{\rm k} < |t|. \end{cases}$$

We evaluate unknown parameters, $a_{\rm k}$, making use of the least square fit. The density, i.e. the number of particles between J and $J+\Delta J$, are plotted as a function of J in Fig. 5. The improvement of uniformity is obvious. It is consequently proved that the peak line density is reduced without emittance blow-up in this method.

A pseudo-uniform distribution has been also obtained by modulating the moving buckets in both the Booster and the Main Ring. For the moving buckets we have to track the synchrotron frequency. The tracking was realized by frequency-modulating the clock of the arbitrary waveform generator, which was accomplished by using another pair of arbitrary waveform generators, one is for generating the voltage proportional to $2f_0$ and the other is used as a voltage controlled oscillator. Due to restricted time, we realized the complete tracking only in the Main Ring. The timing and the degree of modulation were adjusted so as to minimize the beam loss at γ_t, typically starting 100 ms after the beginning of acceleration, continuing ~80 ms and modulating 4%. Figures 6 and 7 show the bunch shapes without/with band-limited white signals that were observed ~15 ms before transition energy. The discrepancy between both cases is clear.

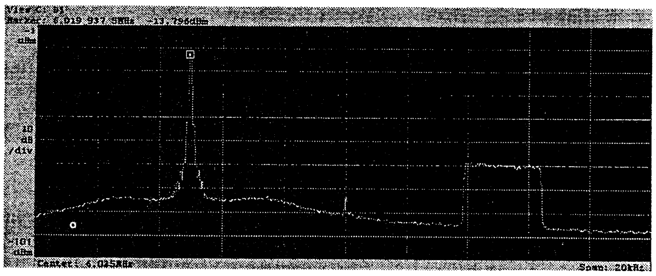

Fig. 2 Frequency spectrum of the rf voltage.

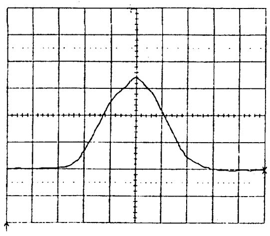

Fig. 3 Bunch profiles without rf voltage modulation, abscissa: 20 ns/div.

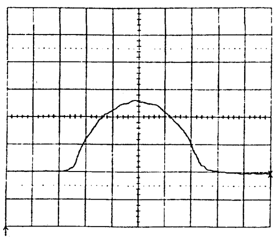

Fig. 4 Bunch profiles with rf voltage modulation. $\xi_{RMS} \approx 3.6\%$, abscissa: 20 ns/div.

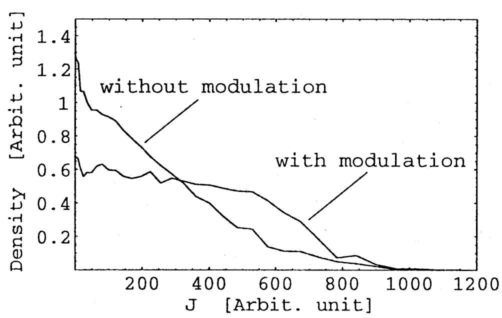

Fig. 5 Phase space densities with/without rf voltage modulation.

In the Booster, tracking was executed by linearly approximating the synchrotron frequency. The improvements were a little different between May run and June run. The reason may be that the momentum distribution in the Booster was varied by different LINAC-to-Booster matching condition or by changed rf voltage program.

In consequence an beam intensity increased about 24% comparing with the previous method [9] in May, 1999. More than the intensity goal of 6×10^{12} ppp has been achieved at the Main Ring flat top and kept since May, 1999, although increase in the longitudinal emittance before γ_t was inevitable for stability at the transition energy. In this achievement additional machine tunings, especially fine tuning of the octupole current to compromize Landau damping for head-tail instability [10] and dynamic aperture, have been indispensable.

4 Conclusion

By applying rf voltage modulation with a band-limited white signal, diffusion occurs in a bounded area of the longitudinal phase space defined by the frequency

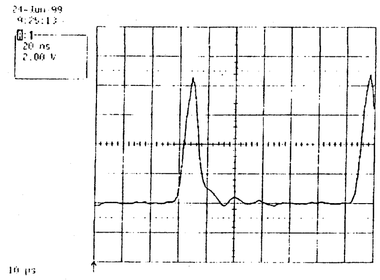

Fig. 6 Bunch profiles without rf voltage modulation, abscissa: 20 ns/div.

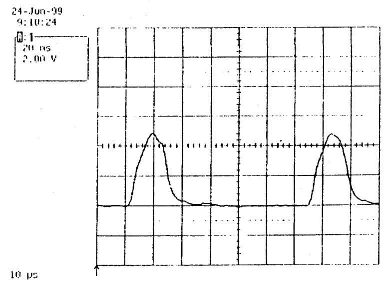

Fig. 7 Bunch profiles with rf voltage modulation. $\xi_{RMS} \approx 4\%$, abscissa: 20 ns/div.

band width.

Demonstration at the Main Ring flat bottom showed that peak line density is reduced without emittance blow-up as theoretically expected.

This method has been successfully applied to the moving rf buckets of the KEK PS, which contributes to achieving the intensity goal of 6×10^{12} protons per pulse.

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